University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
First Exam

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NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, and any notes you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are five problems, each worth 20 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

If you are interested in the ultimate character of the physical world, or the complete world, and at the present time our only way to understand that is through a mathematical type of reasoning, then I don't think a person can fully appreciate, or in fact appreciate much of, these particular aspects of the world, the great depth of character of the universality of the laws, the relationships of things, without an understanding of mathematics. I don't know any other way to do it, we don't know of any other way to describe it accurately ... or to see the interrelationships without it. So I don't think a person who hasn't developed some mathematical sense is capable of fully appreciating this aspect of the world - don't misunderstand me, there are many, many aspects of the world that mathematics is unnecessary for, such as love, which are very delightful and wonderful to appreciate and to feel awed and mysterious about; and I don't mean to say that the only thing in the world is physics, but you were talking about physics and if that's what you're talking about, then to not know mathematics is a severe limitation in understanding the world. —Richard Feynman, *The Pleasure of Finding Things Out*

1. Find $x(t)$ that satisfies

$$\begin{aligned} \dot{x}(t) + 5x(t) &= e^{-5t} \\ x(0) &= 1. \end{aligned}$$

2. Find $x(t)$ that satisfies

$$\begin{aligned} \dot{x}(t) + \frac{5}{t}x(t) &= t \\ x(1) &= 1 \end{aligned}$$

for $t > 0$.

3. Find the general solution to

$$\ddot{x}(t) + 16x(t) = t^2.$$

4. Consider

$$\ddot{x}(t) + \dot{x}(t) - 6x(t) = 1. \tag{1}$$

(a) Use the method of variation of parameters to find the general solution to Equation 1 *Note:* the equation for variation of parameters in the book for second order equations is incorrect. It should be

$$x = c_1 x_1 + c_2 x_2 - x_1 \int \frac{x_2 f}{x_1 \dot{x}_2 - \dot{x}_1 x_2} dt + x_2 \int \frac{x_1 f}{x_1 \dot{x}_2 - \dot{x}_1 x_2} dt,$$

where $x_1(t)$ and $x_2(t)$ are two linearly independent homogeneous solutions.

(b) If

$$\begin{aligned} x(0) &= 1 \\ \lim_{t\to\infty} |x(t)| &< \infty \end{aligned}$$

determine $x(t)$.

5. Consider

$$\ddot{x}(t) + 6\dot{x}(t) + 9x(t) = e^{-3t} + \cos(t). \tag{2}$$

If you were to use the method of undetermined coefficients, which of the following assumptions for the particular solution will, when substituted into Equation 2, yield a set of equations that will allow you to find the particular solution, *i.e.,* which of these assumptions will work? Which assumption is the best?

Explain each answer.

(a)

$$x_p(t) = Ae^{-3t} + B\cos(t)$$

(b)

$$x_p(t) = Ae^{-3t} + B\cos(t) + C\sin(t)$$

(c)

$$x_p(t) = t\left(Ae^{-3t} + B\cos(t) + C\sin(t)\right)$$

(d)

$$x_p(t) = t^2\left(Ae^{-3t} + B\cos(t) + C\sin(t)\right)$$

(e)

$$x_p(t) = t^2Ae^{-3t} + B\cos(t) + C\sin(t)$$

(f)

$$x_p(t) = \left(At^2 + Bt + C\right)e^{-3t} + D\cos(t) + E\sin(t)$$

(g)

$$\begin{aligned} x_p(t) &= t^2\left(A_2e^{-3t} + B_2\cos(t) + C_2\sin(t)\right) + \\ &\quad t\left(A_1e^{-3t} + B_1\cos(t) + C_1\sin(t)\right) + \\ &\quad \left(A_0e^{-3t} + B_0\cos(t) + C_0\sin(t)\right). \end{aligned}$$