University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Control I
Exam 2

B. Goodwine
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NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may only consult the course text and any notes you have written in it. You may **not** use a calculator or any other electronic device.
- There are four problems, each is worth 25 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

A lawyer and an engineer were fishing in the Caribbean. The lawyer said, "I'm here because my house burned down and everything I own was destroyed by the fire. The insurance company paid for everything."
"That's quite a coincidence," said the engineer. "I'm here because my house and all my belongings were destroyed by an earthquake, and my insurance company also paid for everything."
The lawyer looked somewhat confused. "How do you start an earthquake?" he asked.

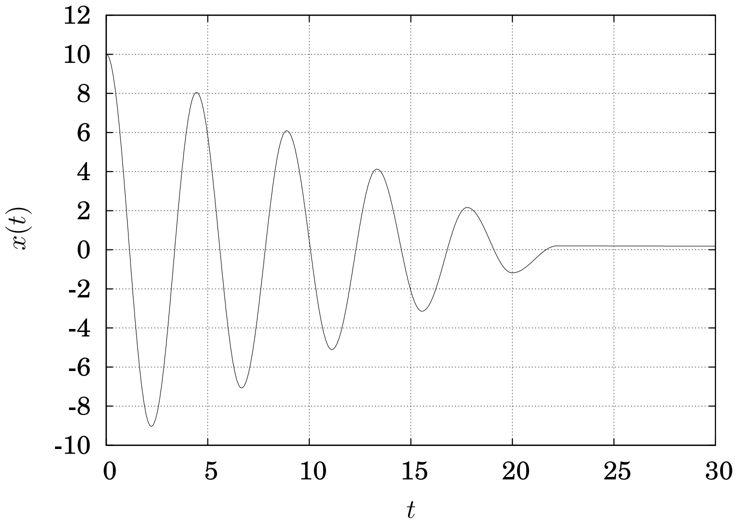

Figure 1. Response for Problem 1.

1. Consider the measured response of a system illustrated in Figure 1. Determine the damping ratio and natural frequency of the system. If you use Figure 1 to obtain some numerical data, be sure to indicate the points you use. You perhaps may find Table 1 on the last page helpful.

2. Consider the mass-spring-damper system illustrated in Figure 4.21 in the course text. If

$$\begin{aligned} m &= 1 \\ b &= 0.2 \\ k &= 10 \end{aligned}$$

and

$$F(t) = \cos t + 6\cos 10t + 3t$$

find $x(t)$ for large t, *i.e.,* the steady state solution. You do not need to find the *exact* mathematical answer. If you can use charts or tables to determine a good approximation, that will suffice.

3. Charles Hermite, pictured in Figure 2, was a French mathematician. The *Hermite equation*, which is very important in quantum physics, is given by

$$\ddot{x}(t) - 2t\dot{x}(t) + \lambda x(t) = 0.$$

Assume a power series to find an approximate solution to

$$\begin{aligned} \ddot{x}(t) - 2t\dot{x}(t) + x(t) &= 0 \\ x(0) &= 1 \\ \dot{x}(0) &= 1. \end{aligned}$$

Grading:

- 13 points are for using the correct approach;
- 2 additional point is for each coefficient in the power series that you determine (up to 6); and,
- 5 points *extra credit* will be awarded if you find the complete solution, *i.e.,* an expression for every a_i in $\sum_{n=0}^{\infty} a_n t^n$ that satisfies the differential equation and initial conditions.

Figure 2. Charles Hermite, December 24, 1822 - 2013 January 14, 1901.

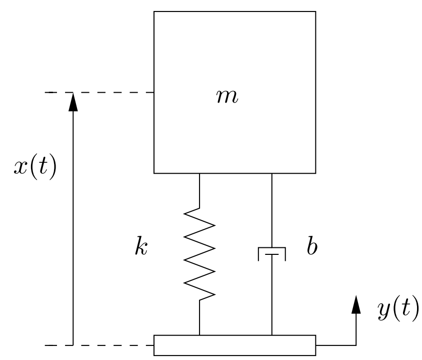

Figure 3. Accelerometer for Problem 4.

4. Figure 3 illustrates a type of accelerometer. It consists of a mass, spring and damper with known characteristics. The base is subjected to some specified motion, $y(t)$. The quantity $y(t)$ is relative to some inertial coordinate system.

   The quantity $x(t)$ is measured relative to the base, *i.e.*, $x(t)$ is a measure of the distance from the base to the mass. This is in contrast to the system covered in class and in the text where $x(t)$ is measured with respect to an inertial coordinate system. In an instrument this make sense: the only sort of thing we can really build can only directly measure how much the mass has moved relative to the case in which it is mounted. You may assume $x = 0$ zero when the spring is at equilibrium.

   Now for an accelerometer, we want to do the inverse of what we did in class. In class we determined $x(t)$ when $y(t)$ was specified. In this problem we know $x(t)$ since we can measure it. What we want to determine is $y(t)$, or in the case of an accelerometer, $\ddot{y}(t)$.

   So, given

$$\begin{aligned} m &= 1 \\ b &= 0.5 \\ k &= 4 \end{aligned}$$

   and

$$x(t) = 3\cos(16t)$$

   determine the magnitude and frequency of $\ddot{y}(t)$.

x	$\ln(x)$
0.05	-3.0
0.1	-2.3
0.15	-1.9
0.2	-1.6
0.25	-1.4
0.3	-1.2
0.35	-1.0
0.4	-0.9
0.45	-0.8
0.5	-0.7
0.55	-0.6
0.6	-0.5
0.65	-0.4
0.7	-0.4
0.75	-0.3
0.8	-0.2
0.85	-0.2
0.9	-0.1
0.95	-0.1
1	0.0
1.05	0.0
1.1	0.1
1.15	0.1
1.2	0.2
1.25	0.2
1.3	0.3
1.35	0.3
1.4	0.3

Table 1. Table of natural logarithms.