University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Control I
Exam 3

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NAME: _____

- You have 50 minutes to complete this exam.
- This is an open book exam. You may only consult the course text and any notes you have written in it. You may also consult a printed version of the chapter on partial differential equations from the updated notes.
- You may use a calculator for computations Problem 2 only. You may not use the calculator for anything other than addition, subtraction, multiplication, division and evaluating trigonometric functions.
- There are three problems. Problems 1 and 2 are worth 35 points each and Problem 3 is worth 30 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

Some people follow their dreams. Others hunt them down and beat them mercilessly into submission.
—*Neil Kendall*

1. Compute the Fourier series for

$$f(x) = \begin{cases} 2, & 0 < x \leq 1 \\ 1, & 1 < x \leq 2 \\ 0, & 2 < x \leq 3 \end{cases}$$

where $f(x+3) = f(x)$. This function is illustrated in Figure 1.

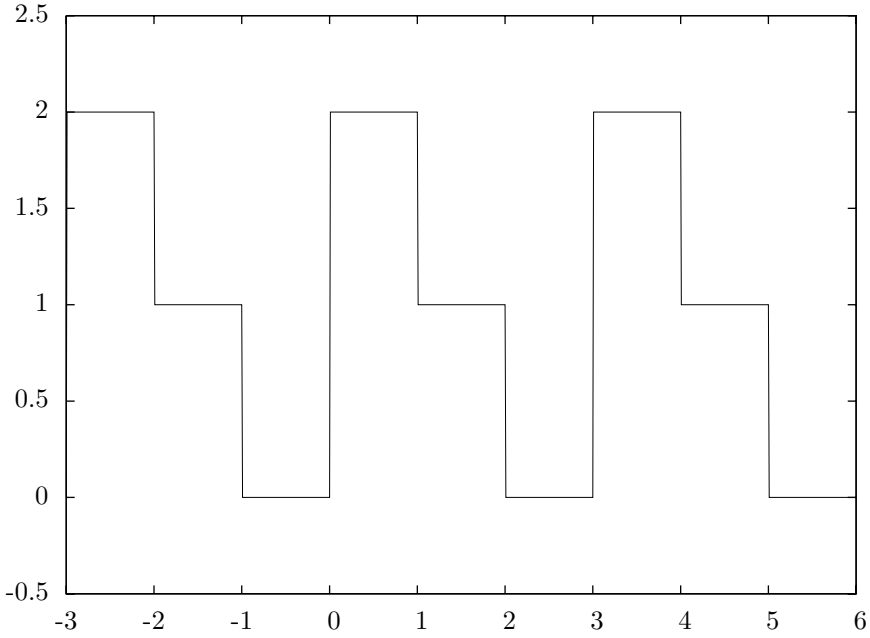

Figure 1. Function for Fourier series in Problem 1.

2. (25 points) Fill in the blanks in the computer program on the next page to complete a program that uses the second order Runge-Kutta method to compute an approximate numerical solution to

$$\begin{aligned} \dot{x} + 0.2x + x^3 &= 0.3\cos t \\ x(0) &= 0. \end{aligned}$$

It is not necessary, but it is permissible to add a function at the end of the program if that is the way you want to do it. Syntax is not the primary criterion for grading this problem, but it does count for part of it.

(10 points) Compute what the first three lines of the output from the program will be. It is permissible to use a calculator for this.

```
      program examprogram

      double precision x,t,dt

      open(unit=13,file="data.d")

      dt = 0.1

      x = ___________________

      do 10 t = 0, 20, dt
         write(13,*) t,x


         x = x + ____________________________________________________________


10    continue
      write(13,*) t,x
      stop
      end
```

3. The partial differential equation that describes the motion of a circular membrane, such as a drum head, is given by the two dimensional wave equation in polar coordinates

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\alpha^2}\frac{\partial^2 u}{\partial t^2}.$$

- (25 points) Assuming that $u(r, \theta, t) = R(r)\,\Theta(\theta)\,T(t)$, find the ordinary differential equations satisfied by $R(r)$, $\Theta(\theta)$ and $T(t)$.

  Hint: since there are three variables instead of two, you must put two variables on one side of the equation and one on the other. This will give you one ordinary differential equation for one variable and an equation for the other two variables. The equation for the other two variables is itself separable, which should give three ordinary differential equations.

- (5 points) Do not actually solve it, but indicate what method you would use to solve the $R(r)$ equation.