University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
First Exam

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NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and four pages of written notes, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems, each worth 25 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

You'll find the more difficulties you have on the way, the more you will enjoy your success.
—*Juha the Cruel Vaatainen*

1. Determine the function $x(t)$ that satisfies

$$\begin{aligned} \ddot{x} + 2\dot{x} + 4x &= 1 \\ x(0) &= 0 \\ \dot{x}(0) &= 0. \end{aligned}$$

Sketch the solution.

2. Determine the solution to

$$\begin{aligned} \ddot{x} + 4\dot{x} + 4x &= e^{-2t} \\ x(0) &= 1 \\ \dot{x}(0) &= 1. \end{aligned}$$

3. Determine the general solution to

$$\left(x - t^2 + te^x\right)\dot{x} + t + e^x = 2tx.$$

4. An airplane is flying to an airport that is located at the origin of a coordinate system. At time $t = 0$ the plane is at the point $(10, 0)$. The pilot attempts to fly to the airport by pointing the nose of the airplane exactly towards the airport at all times. The magnitude of the velocity of the airplane through the air is given by v. The wind is blowing from the south (in the positive y direction) with a magnitude of w. Figures 1 and 2 illustrate the path the airplane follows when $v > w$ and $v < w$ respectively.

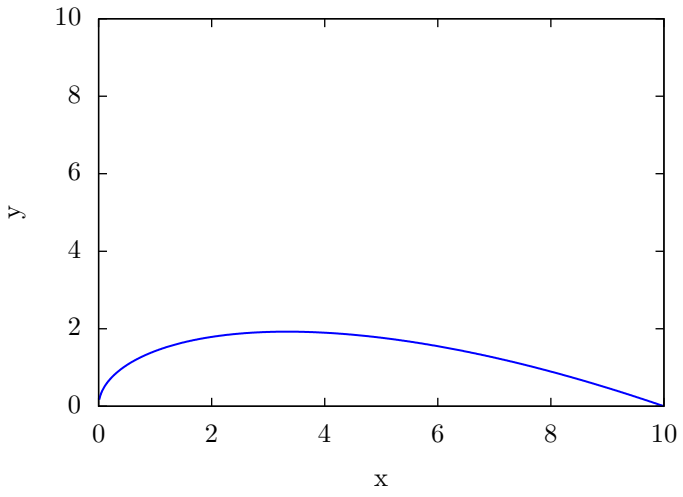

Figure 1. Trajectory of airplane when $v > w$.

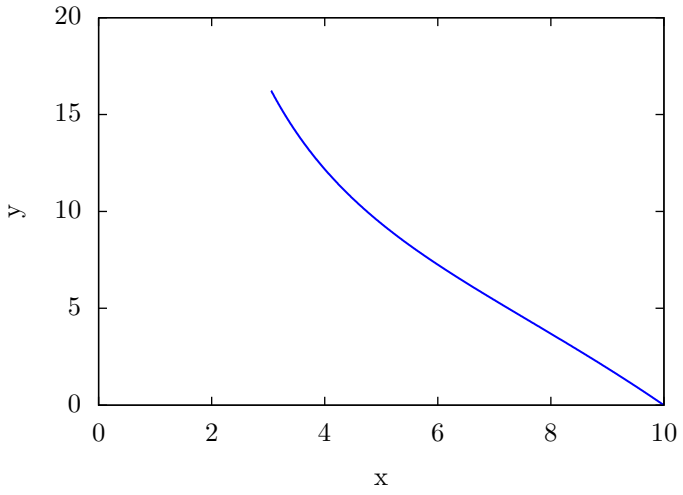

Figure 2. Trajectory of airplane when $w > v$.

(a) Determine the two, first order ordinary differential equations with dependent variables x and y and independent variable t that describe the motion of the airplane, the solution to which is plotted in the figures for different values of v and w. You do not have to solve the equations, just determine what they are.

(b) On the next blank page, write a computer program that will determine an approximate numerical solution to these two first order ordinary differential equations.