UNIVERSITY OF NOTRE DAME
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and four pages of written notes, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems, each worth 25 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem. Especially on Problem 3, if you work is not presented clearly it will be difficult to obtain any partial credit.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

From now on, ending a sentence with a preposition is something up with which I shall not put.
—*Sir Winston Leonard Spencer Churchill*

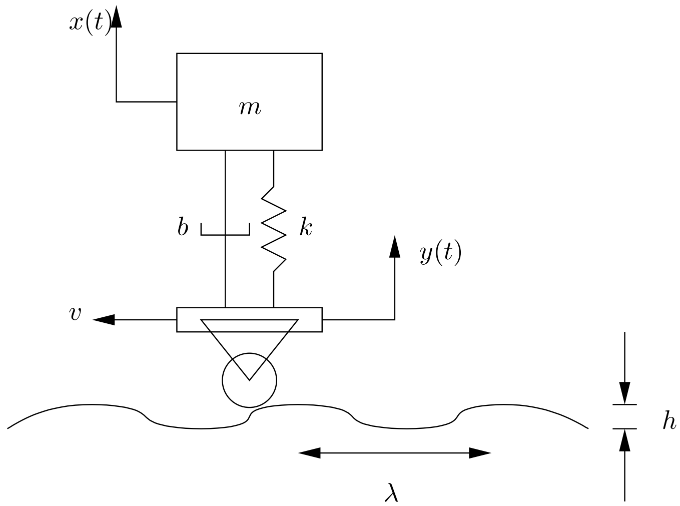

Figure 1. Suspension system for Problem 1.

1. Consider the suspension system illustrated in Figure 1. The squiggly line is supposed to be a sinusoid. Let

$$\begin{aligned} m &= 1 \\ k &= 4 \\ b &= 1 \\ h &= \frac{1}{3} \\ \lambda &= 10 \end{aligned}$$

Figure 2 is the same as Figure 4.20 in the course text on page 117. Figure 3 is what you should have plotted when you did homework problem 4.12.

Explain your answer for each of the following with something like "From Figure 2 if b is incrased, then ζ is ... and that will ... the magnitude of the displacement."

(a) If $v = \frac{10}{\pi}$ should b be increased or decreased to reduce the magnitude of the motion of the mass?

(b) If $v = \frac{10}{\pi}$ should b be increased or decreased to reduce the force exerted on the mass?

(c) If $v = \frac{20}{\pi}$ should b be increased or decreased to reduce the magnitude of the motion of the mass?

(d) If $v = \frac{20}{\pi}$ should b be increased or decreased to reduce the force exerted on the mass?

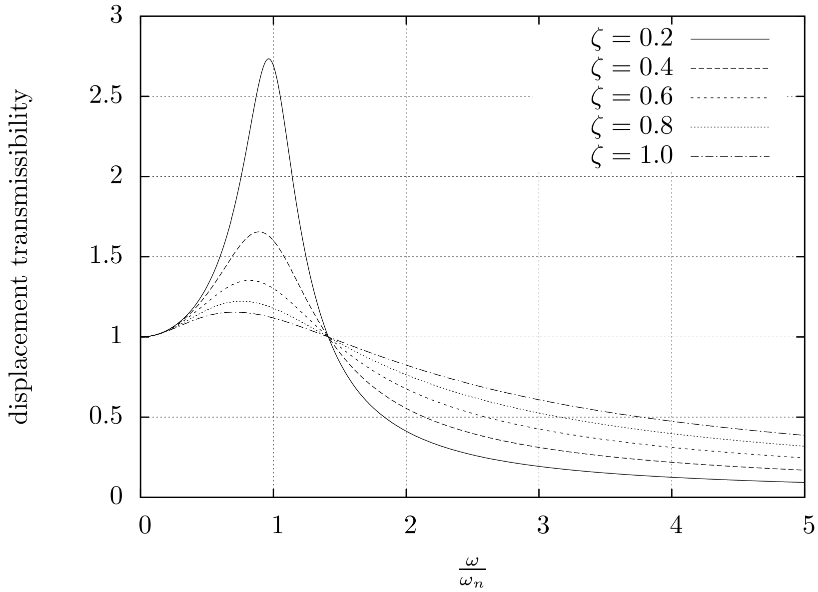

Figure 2. Displacement transmissibility for system in Figure 1.

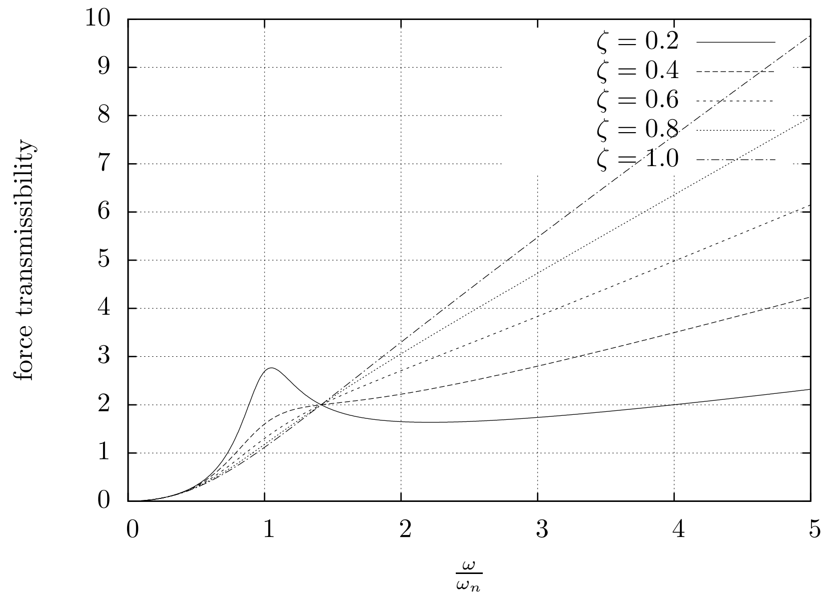

Figure 3. Force transmissibility for system in Figure 1.

2. Determine the first five terms in a series solution about $t = 0$ for

$$\begin{aligned} \ddot{x} + t^2 x &= 0 \\ x(0) &= 1 \\ \dot{x}(0) &= 2. \end{aligned}$$

3. Consider the ordinary differential equation

$$\ddot{x} + x = \sin\left(t^2 x^3 \dot{x}^2\right)$$

and let

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x}. \end{aligned}$$

We want to find an approximate numerical solution using a time step of Δt. Circle the method(s) for which the equation

$$x_1(t + \Delta t) = x_1(t) + x_2(t)\,\Delta t + \frac{1}{2}\sin\left(t^2 x_1^3(t) x_2^2(t)\right)(\Delta t)^2$$

is correct.

(a) Euler's method;

(b) second order Taylor series;

(c) second order Runge-Kutta;

(d) fourth order Runge-Kutta.

You *must* justify your answer for each case. There will be no partial credit for simply guessing.

4. Consider the ordinary differential equation

$$\begin{aligned} \ddot{x} + x &= \sin\left(t^2 x^3 \dot{x}^2\right) \\ x(0) &= 1 \\ \dot{x}(0) &= 2. \end{aligned}$$

Write a computer program that uses the second order Runge-Kutta method to determine an approximate numerical solution. It is permissible to use functions.