University of Notre Dame
Aerospace and Mechanical Engineering

**AME 30314: Differential Equations, Vibrations and Controls I**
**First Exam**

B. Goodwine
September 23, 2009.

NAME:_____

ID Number:_____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems, each worth 25 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

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We're just two lost souls swimming in a fish bowl, year after year

—Pink Floyd, *Wish You Were Here*

1. It is now 2013 and you work for The Boeing Company in the commercial aircraft division. You are responsible for the fabrication of certain composite materials for the fuselage.

   As part of the fabrication process, you encounter the following scenario. A vat contains 100 liters of water. In error someone pours 100 grams of a chemical into the vat instead of the correct amount, which is 50 grams. To correct this condition, a stopper is removed from the bottom of the vat allowing 1 liter of the mixture to flow out each minute. At the same time, 1 liter of fresh water per minute is pumped into the vat and the mixture is kept uniform by constant stirring.

   (a) (15 points) If $x(t)$ represents the number of grams of chemical in the solution at time $t$, the equation governing $x$ is

   $$\begin{aligned} \frac{dx}{dt} &= -\frac{x}{100} \\ x(0) &= 100. \end{aligned}$$

   How long will it take for the mixture to contain the desired amount of chemical?

   It is acceptable to leave your answer in terms of a function, *e.g.,* $t = \sin(35)$ except for simple cases such as $t = \cos(\pi)$ in which case you should write $t = -1$.

(b) (10 points) Determine the equation governing $x(t)$ if the amount of water in the vat is $W$ liters, the rate at which the mixture flows out is $F$ liters/minute (and the same amount of fresh water is added) and the amount of the chemical initially added is $C$.

Little or no credit will be given for guessing. You must derive the equation and/or show why it describes the physical aspects of the described scenario.

2. Determine the general solution to

$$\ddot{x} + 4\dot{x} + 8x = \sin 2t.$$

3. Consider

$$\ddot{x} + 2\dot{x} + 2x = e^{-t} \sin t + 1.$$

(a) Determine the general solution. (15 points)

(b) Sketch the solution if $x(0) = \dot{x}(0) = 0$. (10 points)

4. Determine the solution to

$$\begin{aligned} \dot{x} &= \alpha x \\ x(0) &= 1. \end{aligned}$$

On the same graph, sketch the solution for $\alpha = -2$, $\alpha = -1$ and $\alpha = 0$.