University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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NAME:______________________________

ID Number:______________________________

- Do not start or turn the page until instructed to do so.
- You have 120 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, the new handout of chapter 5, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are three problems. Problem 1 is worth 40 points and the other two problems are worth 30 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

Pretty soon life's little Twinkie gauge is gonna go empty.

—Tallahassee, *Zombieland*

1. You are driving down a sinusoidally-bumpy road in a vehicle that can be modeled like the system illustrated in Figure 4.18 in the course text.

   - The distance between the peaks of the bumps is $\lambda = 10$ m.
   - The mass of the vehicle and everything in/on it is $m = 1000$ kg.
   - The spring constant in the suspension is $k = 9000$ N/m.
   - The constant for the shock absorbers is $b = 2400$ N s/m.
   - The speed you are driving is $\frac{60}{2\pi}$ m/s.

   There are zombies all over the outside of the car that you need to shake off the car by making the magnitude of the shaking as great as possible.

   (a) By referring to Figure 1, which is the same as 4.20 in the course text, do you want to speed up, slow down or maintain your speed? Explain your answer, perhaps by annotating the figure with data from this problem. Answers with no explanation will receive zero points.

   (b) By what factor are you able to maximize the magnitude of the shaking, *e.g.*, 2 times, greater, 10 times greater, no times greater, *etc.*? Explain your answer. At what speed would that happen?

   (c) As is well-known, the zombies are tenacious and after driving down the road for a while the shock absobers are now trashed so b is decreasing. If you are driving at the original speed, will the decreasing b increase or decrease the magnitude of the shaking? If you are driving at the speed which always maximizes the magnitude of the shaking, will the decreasing b increase or decrease the magnitude of the shaking? Explain your answer.

   (d) Attempting to shake the zombies off the car is not working. You decide to switch strategies to liquefy what is remaining of their internal organs by subjecting to as much acceleration (and hence force) as possible. If you are driving at the original speed listed at the beginning of the problem and by referring to Figure 2, which is a graph of what you should have done for part 2 of homework problem 4.12, do you want to increase or decrease your speed? Explain your answer. Also, explain what the best speed to drive would be to maximize the force exerted on the car, and hence the zombies.

   I want you to solve this problem by referring to the relevant figures as indicated. It is possible by solving all the differential equations again using the numbers given. If you do that (correctly) you will recive 75% credit for this probem.

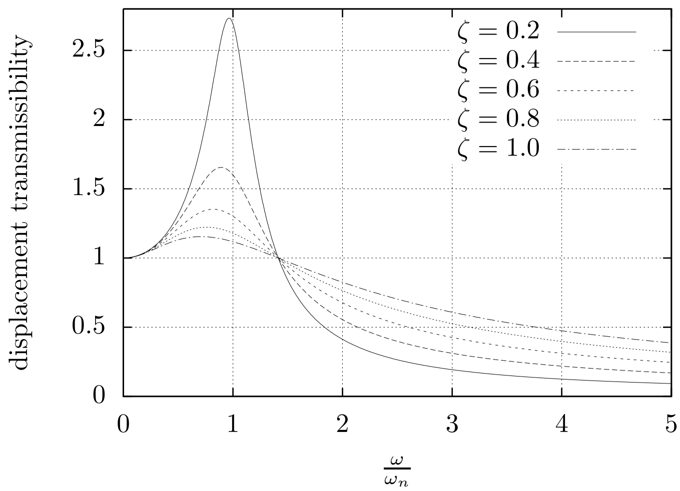

Figure 1. Displacement transmissibility for Problem 1. This is the same figure as Figure 4.20 in the course text.

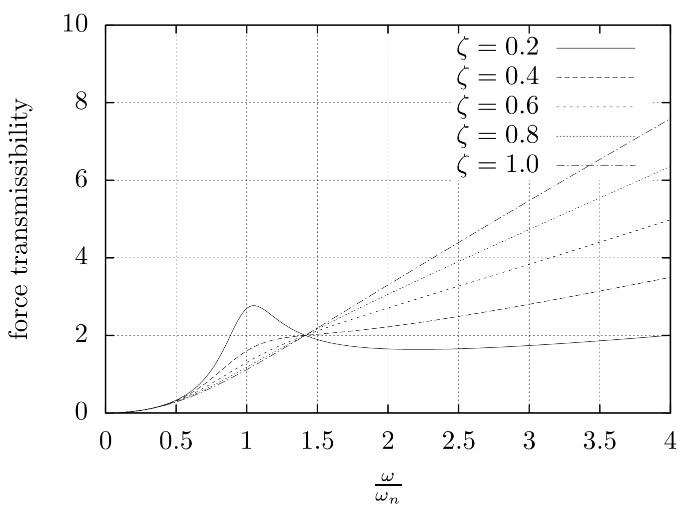

Figure 2. Force transmissibility for Problem 1. This is the same figure you should have made on your own for homework problem 4.12, part 2.

2. The spread of the virus which caused the zombie apocalypse is described by

$$\begin{aligned} \dot{x} &= 3x \qquad (1) \\ x(0) &= 1. \end{aligned}$$

(a) Using a method from Chapter 2 in the course text (or by inspection), determine the solution (*not* a series solution).

(b) Compute the Taylor series for that solution about the point $t_0 = 0$.

(c) Assume a series solution of the form

$$x(t) = \sum_{n=0}^{\infty} a_n t^n$$

and determine the recursion relation for the coefficients a_i. Show that the coefficients a_0 through a_5 the same as the Taylor series from part 2b.

3. You can save the world from the zombies if only you can obtain a good solution to

$$\left(2+t^2\right)\ddot{x}-t\dot{x}+4x=0.$$

Assume a solution of the form

$$x\left(t\right)=\sum_{n=0}^{\infty}a_n t^n$$

and determine the recursion relation for the coefficients a_i. If $x(0)=2$ and $\dot{x}(0)=-3$, find the numerical values for a_0 through a_5.