University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Third Exam

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December 2, 2009.

NAME:______________________________

ID Number:______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and anything you have written in it, but nothing else.
- You may **not** use a calculator or other electronic device.
- There are four problems. Problems 1, 2 and 3 are each worth 30 points, and Problem 4 is worth 10 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

The secret of knowing the most fertile experiences and the greatest joys in life is to live dangerously. —Nietzsche

1. Consider

$$\begin{aligned}
\ddot{x} + \sin\left(xt^2\right)\dot{x} + x^3 &= \cos\left(3\dot{x}t\right) \\
x\left(0\right) &= 2 \\
\dot{x}\left(0\right) &= 3.
\end{aligned}$$

On the *next* page, write a complete FORTRAN (or C, or C++) program that uses the 4th order Runge-Kutta method to determine an approximate numerical solution to this differential equation.

On *this* page, write a program using the same programming language that uses Euler's method to determine an approximate numerical solution to this differential equation. If large blocks of your program are the same, you may indicate them as something like "Block A" on the next page, and then just write "Block A goes here" on this page. The parts of the program that are different must be written out. Points will be deducted for syntax errors, but keep in mind that the main point of the problem is to implement the algorithm, not write perfect code.

2. Compute the Fourier series for

$$f(x) = \begin{cases} 1, & 0 < x \leq \pi \\ 0, & \pi < x \leq 7\pi \\ -1, & 7\pi < x \leq 8\pi \end{cases}$$

where $f(x + 8\pi) = f(x)$.

3. In this problem you will solve what is known as the *interior Dirichlet problem* for a circle. Use separation of variables to show that the solution to

$$\begin{aligned}\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} &= 0\\ u(1,\theta) &= g(\theta)\end{aligned}$$

is

$$u(r,\theta)=\sum_{n=0}^{\infty} r^n\left(a_n\cos n\theta+b_n\sin n\theta\right).$$

Do *not* substitute this solution into the PDE to show it is a solution (it is). Go through the separation of variables process to construct it. Also, provide formulae for the coefficients a_n and b_n.

Hint: you may have to use a clue from the solution provided to determine the solution to the $R(r)$ equation. It is ok to do that.

4. Consider

$$\begin{aligned} \dot{x} &= 1 \\ x(0) &= 0. \end{aligned}$$

Of the three methods,

(a) Euler's method;

(b) the third order Taylor series method; and,

(c) 4th order Runge-Kutta method;

for a given $\Delta t \ll 1$, which would determine an approximate numerical solution with the least error? Explain your answer.