University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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ID Number:_____

NAME:_____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else. You may have supplemented your notes with your own summaries or hand-written copies of other material, but you may not consult printed homework solutions, photocopies of homework solutions, *etc.*
- You may **not** use a calculator or other electronic device.
- There are four problems. Each problem is worth 25 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- In the opinion of the instructor, the first three problems are like the homework exercises, but the fourth problem will require building on the knowledge obtained from doing the homeworks.
- If you do not have a stapler, do not take the pages apart.

There are certain queer times and occasions in this strange mixed affair we call life when a man takes this whole universe for a vast practical joke, though the wit thereof he but dimly discerns, and more than suspects that the joke is at nobody's expense but his own.

—Herman Melivlle, *Moby Dick*

1. Determine a solution to

$$\begin{aligned} \ddot{x} + \frac{1}{1+t^4}x &= 0 \\ x(0) &= 2 \\ \dot{x}(0) &= 3. \end{aligned}$$

2. Write a computer program in FORTRAN, C or C++ to determine an approximate numerical solution to the differential equation from Problem 1.

   The problem will mainly be graded looking for the correct implementation of the algorithm. Only glaring syntax mistakes will result in point reductions.

3. Assume a series solution about $t = 0$ to determine a solution to

$$t^2\ddot{x} + x = 0.$$

4. (a) (9 points) Write a computer program in FORTRAN, C or C++ that determines an approximate numerical solution to the equation from Problem 1 using the second-order Taylor series method.

   If you do not want to, you do not have to re-write all the "program taylor" "real this" "integer that" stuff. You can just refer back to your program from Problem 2, mark an important section with (*) and here put "change (*) to ...," "remove (**)" *etc.* Or you can just write the whole program, it is your choice.

(b) (8 points) Show that for the differential equation

$$\dot{x} = \sqrt{x}$$

all methods that are second-order and higher will give the same answer, the exact solution.

(c) (8 points) For what range of values of the independent variable will the series solution to the differential equation in Problem 1 converge? Justify your answer by, for example, proving it, referring to theorems in the text, *etc.*