University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
First Exam

B. Goodwine
September 14, 2012

ID Number:______________________________

NAME:________________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else. You may have supplemented your notes with your own summaries or hand-written copies of other material, but you may not consult printed homework solutions, photocopies of homework solutions, handouts from other courses, *etc.*
- You may **not** use a calculator or other electronic device.
- There are four problems. Each problem is worth 25 points. The problems get progressively better through the exam.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

A pessimist sees the difficulty in every opportunity; an optimist sees the opportunity in every difficulty.

— Winston Churchill

1. • Determine the general solution to

$$\dot{x} = -2x.$$

• Determine the general solution to

$$\dot{x} + 2x = e^{-2t} + t.$$

2. Determine the solution to

$$\ddot{x} + b\dot{x} + x = 2.$$

If $x(0) = 1$, $\dot{x}(0) = 0$, sketch the solution (x vs. t) for three cases:

- small positive b
- large positive b
- negative b

3. It is now 2016 and you work for Youtube. Due to the unprecedented long-lasting popularity of Psy's *Gangnam Style* your company has decided to launch its own satellite to increase bandwidth to mobile devices.

   Because it is in space, the only way to cool a satellite is for it to radiate energy, primarily through surfaces called radiators (naturally). The famous *Stefan Boltzman law* says that the rate at which energy is radiated from a body is proportional to the temperature to the fourth power, *i.e.*,

$$q = \epsilon \sigma T^4$$

   where ϵ and σ are positive physical parameters, q is the rate at which energy is radiated *per unit area* and $T(t)$ is the temperature.

   (a) The time rate of change of temperature of a body is proportional to the rate of energy transfer into and out of it. Assuming the only energy transfer is radiation out of the body from radiators with surface area A, what is the differential equation describing temperature as a function of time, $T(t)$?

(b) If $T(0) = T_0$, what is the solution to the equation?

(c) You are asked to go to lunch with Psy one day when he is visiting Youtube. He claims that if the surface area of the radiators is doubled, the time it will take for the temperature of the satellite to be reduced in half will be cut in half compared to the time it takes with the original radiators. Your boss says that it will be reduced, but not by a factor of 2, but rather by a factor of $\sqrt{2}$. You decided to cut to the chase and give the real answer based on your analysis so that the lunch conversation can move on to more important matters such as dance moves. What is your analysis and what does it say about how much faster the body will cool if the radiators are doubled in surface area?

4. n Later at lunch, Psy says "You know, I remember from my differential equations course in college that one way to solve a nonlinear, first-order, ordinary differential equation is to check whether it is exact. I've always wondered if there is such a thing as an exact second-order, ordinary, nonlinear differential equation."

   "I'm glad you brought that up," you respond...

   Assume $\psi(x,t) = c$ gives the function $x(t)$ implicitly that is a solution of a second-order, ordinary differential equation. For first-order equations, we computed

$$\frac{d\psi}{dt}(x,t) = \frac{\partial\psi}{\partial x}(x,t)\frac{dx}{dt}(t) + \frac{\partial\psi}{\partial t}(x,t) = 0$$

   and used the equality of mixed partials to determine a test for exactness.

   For this problem, compute

$$\frac{d^2\psi}{dt^2}(x,t) = 0$$

   to determine a form for an exact second-order ordinary differential equation. From that, can you determine a test for exactness for second-order nonlinear ordinary differential equations?