University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I Second Exam

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ID Number:______________________________

NAME:________________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else. You may have supplemented your notes with your own summaries or hand-written copies of other material, but you may not consult printed homework solutions, photocopies of homework solutions, *etc.*
- You may **not** use a calculator or other electronic device.
- There are four problems. Problems 1 through 3 are worth 30 points each. Problem 4 is worth 10 points.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

I am, therefore I must be, I think.

— Pink Floyd

1. Consider a mass-spring-damper system illustrated in Figure 4.1 in the course text, and assume m, b and k are such that the equation of motion is given by

$$\ddot{x} + \frac{1}{10}\dot{x} + x = 1$$

($f(t) = 1$).

   (a) Determine the general solution. It is acceptable to use any formula from the book or notes, as long as you refer to it by number (or date if from notes). You do not have to derive any expressions that you can look up.

   (b) If $x(0) = 0$ and $\dot{x}(0) = 0$ sketch the solution.

   (c) If k is increased, sketch the solution. Point out particularly the differences, *e.g.*, it oscillates faster, decays slower, *etc.* and explain your reasoning.

2. This problem will do *part* of Exercise 4.27 on page 159, which considers the effect of a rotating imbalance on a system. An example of this would be a turbine engine mounted on a wing.

   Take it as *given* that the force exerted by the imbalanced rotating part is

   $$f(t) = m_e r_e \omega^2 \cos \omega t, \tag{1}$$

   where m_e is the mass of the imbalance, r_e is the length of the eccentricity of the imbalance and ω is the angular velocity of the imbalance.

   Consider this force from Equation 1 acting on the mass-spring-damper system illustrated in Figure 4.4 (undamped) of the text.

   (a) Determine the particular solution, x_p (don't worry about the homogeneous solution).

   (b) Manipulate x_p to be of the form

   $$x_p = \frac{m_e r_e}{m} M \cos \omega t$$

   Plot $|M|$ vs frequency ratio, ω/ω_n.
   Explain why the plot make sense.

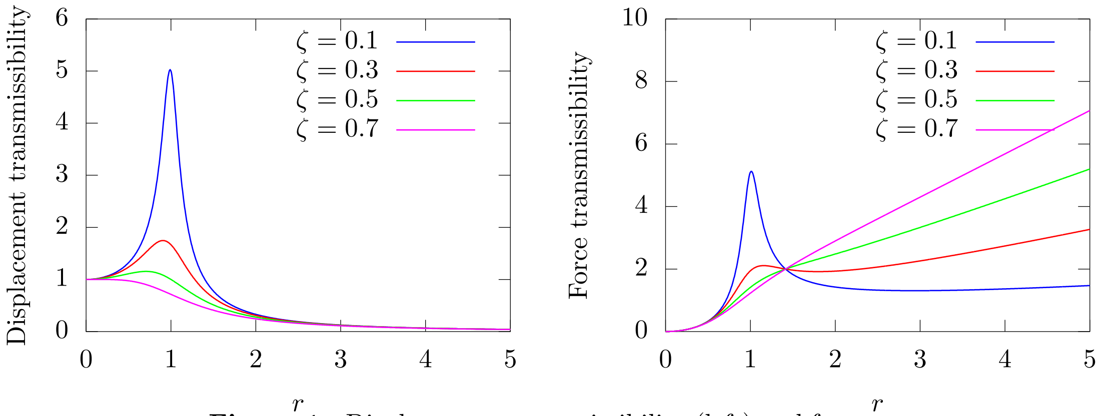

Figure 1. Displacement transmissibility (left) and force transmissibility (right) for suspension system illustrated in Figure 4.18.

3. There is a modern class of fluids called “smart fluids” which have a bunch of ferrous/magnetic particles suspended in the fluid. What makes these “smart” is that a magnetic field imposed on the fluid will affect the behavior of the particles, which changes the viscosity of the fluid.

   This allows us to have dampers, and in particular shock absorbers, for which we can control b by changing the magnetic field around the fluid in the shock absorber.

   Consider the “car driving down the bumpy road” problem we have studied. The displacement- and force-transmissibility plots for that problem are reproduced in Figure 1.

   Consider three cases: slow driving, medium driving (near resonance) and fast driving, and assume that we have a suspension with $\zeta = 0.5$. For all three cases considering *both* force and displacement transmissibility, indicate if we can change b if it's best to increase or decrease b. Explain your reasoning.

   (a) Slow driving: increase/decrease B.

   (b) Medium driving: increase/decrease B.

   (c) Fast driving: increase/decrease B.

4. Chain rule!

(a) If

$$f(x) = e^{\sin(x^2)}$$

compute df/dx.

(b) If we consider the previous function $f(x)$ as the composition of three functions, $f(g(h(x)))$, the chain rule states

$$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dh}\frac{dh}{dx}.$$

Copy your answer from the first part and label which of the terms correspond to df/dg, dg/dh and dh/dx.

(c) For

$$\dot{x} = f(x,t) = x^2 + x\sin t$$

give an expression for $df/dt(x,t)$.