University of Notre Dame
Aerospace and Mechanical Engineering

**AME 30314: Differential Equations, Vibrations and Controls I Third Exam**

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ID Number:_____

NAME:_____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else. You may have supplemented your notes with your own summaries or hand-written copies of other material, but you may not consult printed homework solutions, photocopies of homework solutions, *etc.*
- You may **not** use a calculator or other electronic device.
- There are three problems. Problems 1 and 3 are each with 35 points and Problem 2 is worth 30 points. In the opinion of the instructor Problem 2 requires some extra thought so you should probably do that problem last.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- If you do not have a stapler, do not take the pages apart.

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*The pessimist sees difficulty in every opportunity. The optimist sees the opportunity in every difficulty.*

— Winston Churchill

1. Consider

$$\ddot{x} + \frac{3}{t^2+4}\dot{x} + t^2 x = 0. \tag{1}$$

   (a) Determine a series solution about $t = 0$ for Equation 1.

   (b) For what values of $t$ will this series converge to the solution?

   (c) If the series is computed about $t = 2$, for what values of $t$ would you expect the series to converge to the solution?

2. In all the numerical methods we considered in this class, a *fixed* step size was used. In other words, once you chose `dt` for your problem, it was fixed for all time unless you decided to change it and run the program again.

   There are many *adaptive* step size algorithms that change `dt` as the program is running and in this problem you are going to propose your own. The idea is at each step to evaluate either what `dt` should be or whether it should be increased or decreased for the next step. The goal is to have large step sizes when possible and small step sizes when necessary to balance how long the program takes to run against the error in the numerical approximation.

   In other words, in between `do t=1,tfinal,dt` and `enddo` you should be changing `dt`.

   Consider Euler's method.

   (a) In not too many words, describe the idea behind your adaptive step size algorithm.

   (b) For the differential equation

   $$\begin{aligned} \dot{x} &= tx^2 + \sqrt{x} \\ x(0) &= 2 \end{aligned}$$

   write a program that implements your method.

   (c) For homework extra credit, try your program out after the test and submit it with the next homework.

3. Determine a solution to

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

where

$$\begin{aligned} u(0,t) &= 0 \\ u(3,t) &= 0 \\ u(x,0) &= \begin{cases} 1, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0. \end{aligned}$$