University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
First Exam

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ID Number:______________________________

NAME:__________________________________

GRADE PROBLEMS:______________________________

- Do not start or turn the page until instructed to do so. You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your own course notes, your homeworks, homework solutions, other books, *etc.*
- You will be graded on four of the five problems. You can choose which problem to not have graded. Indicate above which problems you want graded, *e.g.*, "Grade problems: 1, 2, 4 and 5." If you don't indicate which problems you want graded they will be averaged.
- You may **not** use a calculator or other electronic device. You may use a slide rule, abacus or other mechanical computational device.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary. If you do not have a stapler, do not take the pages apart.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation from the book is totally fine as long as it answers the problem.

Ever since I was a child I have had this instinctive urge for expansion and growth. To me, the function and duty of a quality human being is the sincere and honest development of one's potential.
— Bruce Lee

1. Find the general solution (homogeneous + particular) to

$$\dot{x} + \alpha x = \sin \omega t.$$

Make a qualitative sketch of the solution for the cases where α is positive and when it is negative.

2. Determine the general solution to

$$\dot{x} + 2x = t + e^{-2t}.$$

3. Determine the general solution to
$$\dot{x} + tx = 0.$$

4. Fact: For objects moving through the a fluid medium such as the atmosphere, the drag on the object is proportional to the velocity for small velocities and is proportional to velocity squared for large velocities.

   (a) What is the differential equation describing the velocity of a falling object under the influence of gravity for small velocities?

   (b) What is the differential equation describing the velocity of a falling object under the influence of gravity for large velocities?

   (c) For each equation, indicate whether it is linear or nonlinear.

   (d) Don't actually solve them, but for each equation, indicate how you would solve it.

   (e) The *terminal velocity* is the velocity at which the falling object no longer is accelerating. For both equations, what is the terminal velocity? Hint: you can determine this without solving the differential equations.

5. Conisider the vector space of real numbers, $\mathbb{R}$. Consider the operator $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = 0,$$

i.e., whatever you give to f it gives back zero. Is f linear or nonlinear? Justify your answer.

Consider instead

$$f(x) = 1.$$

Is this f linear or nonlinear? Again justify your answer.