UNIVERSITY OF NOTRE DAME
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
First Exam

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ID Number:______________________________

NAME:_________________________________

- Do not start or turn the page until instructed to do so. You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your own course notes, your homeworks, homework solutions, other books, *etc.*
- There are four problems, each worth 25 points.
- You may **not** use a calculator or other electronic device.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary. If you do not have a stapler, do not take the pages apart.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation from the book is totally fine as long as it answers the problem.

The music of the heavens being eternal, Leonardo [da Vinci] understood that friction is absent from the state of grace. Thus confined to this mortal world, friction is a consequence of original sin.

— *Control of Machines With Friction*, by Brian Armstrong-Hélouvry, Springer, 1991, p. 1.

1. Determine the general solution to

$$\ddot{x} + 2\dot{x} + x = t.$$

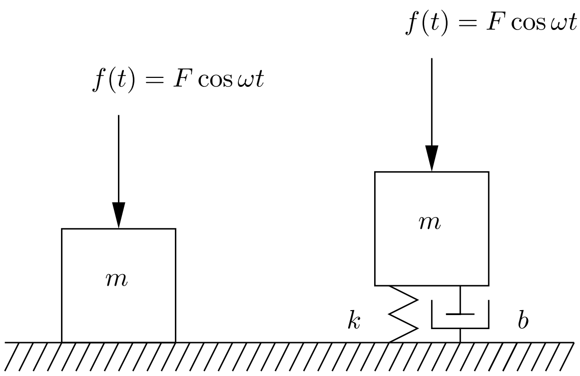

Figure 1: Satellite component for Problem 2.

2. It is now 2018 and you work for Honeywell designing satellite systems. Inside the satellite you are working on there is a box that is subjected to a force as illustrated in Figure 1. Because the manner in which the force is generated, it is accurate to consider it to be harmonic:

$$f(t) = F\cos\omega t,$$

but the frequency is *unknown*.

Your goal is to minimize the magnitude of the force to which the frame of the satellite is subjected. The frame is the part that is drawn as the "ground" in the figure. In other words, you want the ground in the figure to only feel a small force.

If the box is bolted to the satellite frame, it will look like the left part of Figure 1, and your group is discussing the efficacy of attempting to isolate the device using a spring and damper as is illustrated on the right.

Determine the force to which the frame is subjected for both the left and right cases illustrated in the Figure (consider the cases separately – there are not two boxes at the same time). In order to clearly communicate to your boss the nature of your analysis, write it as a "transmissibility" problem in which you present the answer in the form

$$f_g(t) = FM\cos(\omega t + \phi)$$

where f_g is the force on the ground from the box. Sketch M versus frequency ratio, ω/ω_n. Does the spring/damper always help, never help or sometimes help? If the answer is sometimes, when does it help?

You must include all the computations for this problem. Simply saying it's just like the derivation on page 1234 in the book is not acceptable (but of course if you think it's just like that, then parallel the development...)

3. Consider

$$a\ddot{x} + b\dot{x} + cx = 0$$

and assume that none of the a, b or c are zero and the initial conditions are not zero.

(a) Show that if a, b and c have the same sign, then

$$\lim_{t\to\infty} x(t) = 0.$$

(b) Show that if one of the three coefficients has a different sign than the other two then

$$\lim_{t\to\infty} x(t) = \pm\infty.$$

You do *not* have to consider the case of repeated roots. In words, show that if all the coefficients have the same sign, then the solution is stable; otherwise, the solution is unstable.

4. You have not seen this problem before and it's not in the book. I would suggest spending a few minutes getting started on this but not spending any more time than that before you know you earned a good chunk of the credit from the other problems.

   Consider the *nonlinear* initial value problem

$$\begin{aligned} \ddot{x} + x + \epsilon x^3 &= 0 \\ x(0) &= 1 \\ \dot{x}(0) &= 0 \end{aligned}$$

   where ϵ is a small constant. This is nonlinear, but unfortunately, "We have not covered a method to solve this" is not an acceptable answer to this because this problem will work you through a way to do it.

   One way to solve this is to assume

$$x(t) = x_0(t) + \epsilon x_1(t) + \epsilon^2 x_2(t) + \cdots \tag{1}$$

   where ϵ is the same constant as in the equation and all the functions $x_0(t)$, $x_1(t)$, $x_2(t)$ need to be determined.[1]

   The approach is to take the solution from Equation 1 and substitute it into the differential equation to get separate equations for x_0, x_1, *etc.* by considering separately the terms multiplying ϵ^0, ϵ^1, ϵ^2, *etc.* and thinking of the 0 on the right-hand side as $0 + \epsilon 0 + \epsilon^2 0 + \cdots$. Cubing the series is difficult, but it is not too hard to figure out the first few terms.

   Do that substitution. What is multiplying $\epsilon^0 = 1$? What is multiplying ϵ^1? Solve the first one and use the initial conditions. Then solve the second one.

[1] This is a series, but it's not the same sort of series solution we are studying in Chapter 5. This approach belongs to the class called *perturbation methods*.