University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Third Exam

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ID Number:______________________________

NAME:_________________________________

- Do not start or turn the page until instructed to do so. You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your own course notes, your homeworks, homework solutions, other books, *etc.*
- There are three problems, each worth 33 points (one free point!).
- You may **not** use a calculator or other electronic device.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary. If you do not have a stapler, do not take the pages apart.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation or using an equation or graph from the book is totally fine as long as it solves the problem. In fact the exam time is calibrated with the expectation that you solve the problem in the most efficient manner possible.

Did you hear about the man who got cooled to absolute zero?

He's 0K now.

1. Consider the system illustrated in Figure 1. Determine $x(t)$ if $x(0) = 1$ and $\dot{x}(0) = 0$.

   Assume that the part joining the springs that displaces by y remains vertical and has no mass.

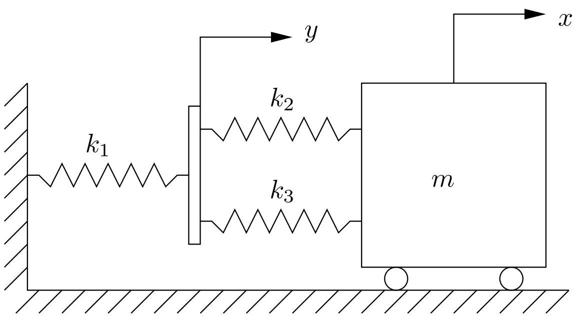

Figure 1: System for Problem 1.

   What is the natural frequency of the system? Would it be correct to use the magnification factor plot in the book (such as Figure 4.5) using this natural frequency if the system were forced?

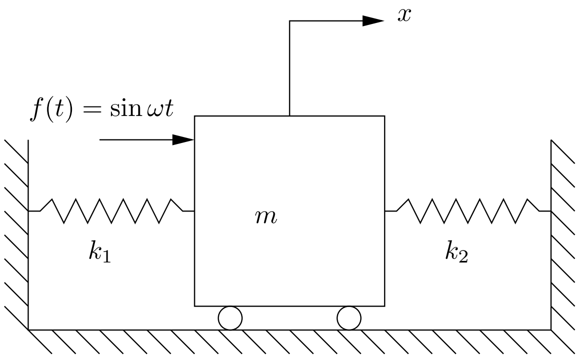

Figure 2: System for Problem 2.

2. Consider the system illustrated in Figure 2 with $k_1 = k_2 = 32$, $m = 1$ and $\omega = 7$. The goal is to *decrease* the force transmitted to the *left* wall in *steady-state* by a simple redesign of the system. You don't care at all about the force on the right wall, so it may increase or decrease. Because you are concerned with the steady-state solution, only consider the particular solution.

   You only have three options:

   (a) decrease k_1 to $k_1 = 17$

   (b) increase k_1 to $k_1 = 49$ or

   (c) leave the design the same.

   Which is the best option to decrease the force on the left wall? Justify your answer.

   If you choose not to do some long division but otherwise do the analysis correctly (e.g., "Option 1 is best as long as $22(35)/54 < 11(56)/123$, and if not option 3 is best."), you will lose only 2 points.

   If you were able to change k_2 by a small amount, would increasing or decreasing it decrease the force on the left wall?

   Extra credit (10 points): Compute the derivative of the force on the wall with respect to k_1 (use the numerical values for the other parameters). Evaluate the derivative at $k_1 = 32$ and interpret the result.

3. Determine the solution to

$$\begin{aligned} \ddot{x} + \frac{1}{t+3}\dot{x} &= 0 \\ x(0) &= 1 \\ \dot{x}(0) &= 2. \end{aligned}$$

Even if this is worked out in the book somewhere (I can't remember), you must work through the solution method. For what values of t will your solution converge?