University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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October 16, 2014

- Do not start or turn the page until instructed to do so. You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else.
- There are five problems, each worth 20 points.
- You may **not** use a calculator or other electronic device. You may use an electronic reading device if you have obtained approval from the instructor to do so.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation from the book is totally fine as long as it answers the problem.

Led Zeppelin took the blues, gave it a lobotomy, ran about 40 million volts of current through it, and then set it loose on an unsuspecting public.

— Review of *Black Dog* on American Hit Network

1. Consider the mass-spring system illustrated in Figure 1 with $m = 1$, $k_1 = k_2 = 8$ and $f(t) = \cos 2t$. If k_2 is slightly increased, what is the effect on

   (a) the magnitude of the motion of the mass

   (b) the magnitude of the total force experienced by the mass

   (c) the magnitude of the force on the left wall

   (d) the magnitude of force on the right wall?

   Only consider steady-state and the particular solution.

   If you are free to choose a value for k_2 anywhere between 1 and 16, what value would minimize the magnitude of the force on the left wall?

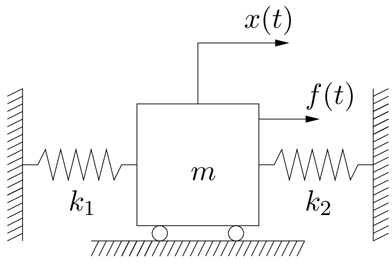

Figure 1: Mass-spring system for Problem 1.

2. Consider the system illustrated in Figure 2 with *two* springs connected in series. The point of this problem is to determine what a single equivalent spring would be.

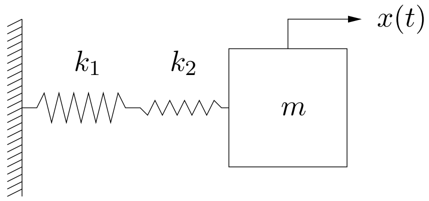

Figure 2: Two-spring system for Problem 2.

   If the springs are in series, they both will be subjected to the same force. To determine an equivalent spring force, do the following:

   - Determine how much each spring will be deflected under a static force of magnitude F.
   - Adding these two deflections together will give the deflection of the two-spring system under a force F. Since you now know both the load, F and the deflection, you can find an equivalent single spring with constant k_{equiv}. What is it?

- If you double one of the spring constants, how will that change the frequency of oscillation of the system in Figure 2?

3. Consider the graphs in Figure 3. All of them illustrate two solutions with initial conditions $x(0) = \dot{x}(0) = 1$. You will receive half credit for the correct match and half credit for providing an explanation.

   (a) Match the solutions in the upper left figure to

   i. $2\ddot{x} + \dot{x} + 8x = 0$

   ii. $2.5\ddot{x} + \dot{x} + 8x = 0$.

   (b) Match the solutions in the upper right figure to

   i. $3\ddot{x} + \dot{x} + 8x = 0$

   ii. $3\ddot{x} + \frac{1}{2}\dot{x} + 8x = 0$.

   (c) Match the solutions in the lower left figure to

   i. $\ddot{x} + \dot{x} + 8x = 0$

   ii. $\ddot{x} + \dot{x} + 14x = 0$.

   (d) Match the solutions in the lower right figure to

   i. $\ddot{x} + \dot{x} + 8x = 0$

   ii. $2\ddot{x} + 2\dot{x} + 25x = 0$.

4. In Exercise 4.27 from the course text, you determined the steady-state solution for the motion of a mass mounted on a spring that had an unbalanced motor attached to it. If you did it correctly, you found the particular solution to be

$$x_p(t) = r\frac{m_e}{m}\left[\frac{\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]\cos\omega t. \tag{1}$$

   Derive a similar expression for the steady-state solution if a damper is added in parallel with the spring. Put it in the form of

$$x_p(t) = r\frac{m_e}{m}M\cos\left(\omega t + \phi\right)$$

   and plot $|M|$ versus frequency ratio for a few values of ζ.

5. Determine the solution to $\ddot{x} + 2\dot{x} + x = te^{-t}$ where $x(0) = 1$ and $\dot{x}(0) = 1$.

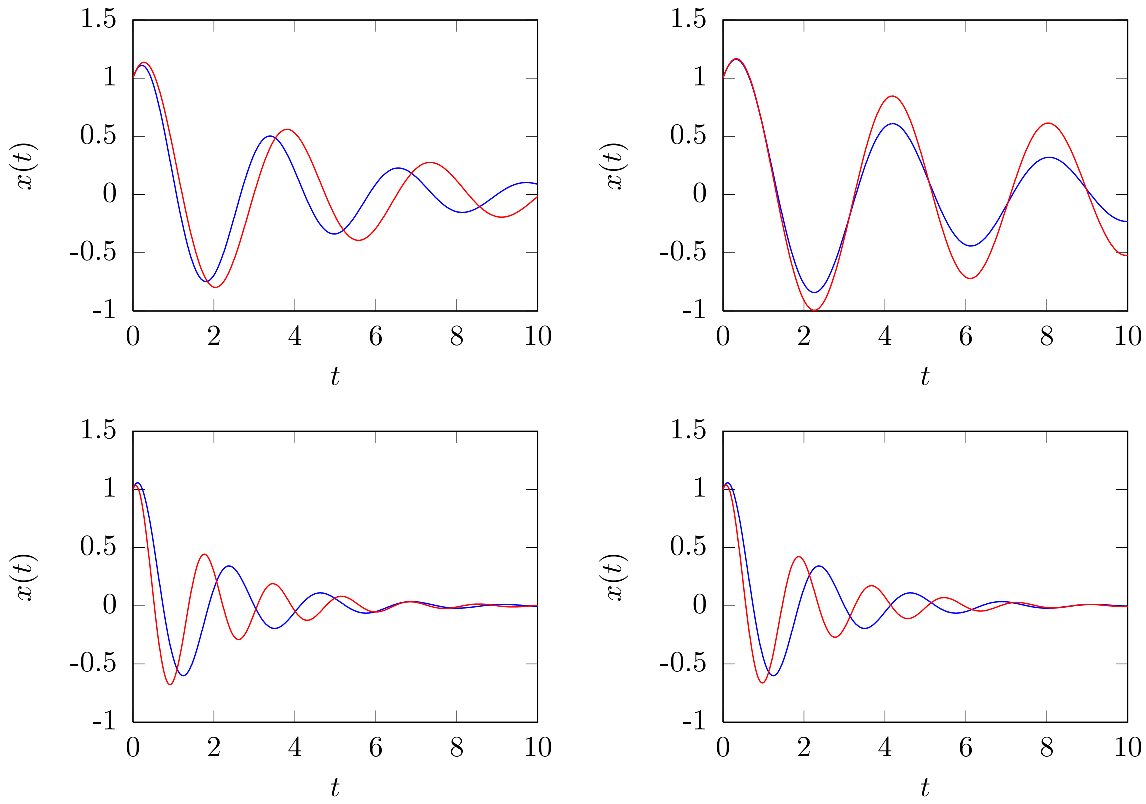

Figure 3: Figures for Problem 3.