University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Third Exam

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- Do not start or turn the page until instructed to do so. You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else.
- Problems 1 and 2 are worth 30 points each, and Problem 3 is worth 40 points. Problems 1 - 3 are required.
- Problem 4 has three extra credit problems. The number of points each is worth depends on how many people do these. Unlike the partial credit for the required problems, barely starting or saying how you would solve the problem without actually doing it won't be worth any credit. So, don't bother unless you have finished the rest of the exam and have 10 minutes or so to make some serious progress. If you have less than 10 minutes, you will be better off double-checking the answers to the required problems.
- You may **not** use a calculator or other electronic device. You may use an electronic reading device only if you had obtained approval from the instructor to do so on Exam 2 and only use it to refer to the course text or your own notes.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.
- You may choose whatever method you like to solve the problems unless the problem specifies which method to use. Merely substituting into an equation from the book is totally fine as long as it answers the problem.

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1. Consider $\dot{x} + 3x = 0$ where $x(0) = 1$.

   (a) Determine a power series solution for this equation by assuming a solution of the form $x(t) = \sum_{n=0}^{\infty} a_n t^n$ and determining the coefficients a_n.

   (b) For what range of t would you expect the series to converge to the solution?

   (c) Determine the solution using a method from Chapter 2 in the course text and compute a Taylor series for that solution about $t = 0$. Is the series the same as you determined in Part 1a of this problem? You may show they are the same by showing the first 5 terms in the series are the same (or find a series expression for each one).

2. Determine the general solution to

$$\ddot{x} + (t + 5)\,\dot{x} + 3x = 0.$$

3. Determine the solution to

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \frac{1}{10}\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial^2 x}(x,t)$$

   where $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = g(x)$. Assume $L < 20\pi$ (credit will be given for showing where this is needed). What sort of physical system would this represent?

4. Extra credit problems:

   (a) Do Exercise 11.14 from the book.

   (b) Solve the partial differential equation given in Exercise 11.14 in the book (you may get credit for solving it even if you don't derive it in the immediately-preceding problem).

   (c) Do Exercise 11.21 from the book.