University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Second Exam

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October 13, 2016

- Do not start or turn the page until instructed to do so. You have 75 minutes to complete this exam.
- This is a closed book exam.
- There are four problems, each worth 25 points.
- You may **not** use a calculator or other computational aid.
- Your grade on this exam will constitute 15% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.
- You may use the information provided on Page 2 in any problem. However, it is not required that you do so.

You say the hill's too steep to climb
Chiding
You say you'd like to see me try
Climbing
You pick the place and I'll choose the time
And I'll climb
The hill in my own way
Just wait a while, for the right day
And as I rise above the treeline and the clouds
I look down hearing the sound of the things you said today

—Pink Floyd, *Fearless*

Some Results from Chapter 4: For $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F\cos(\omega t)$ which can be written as $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \delta\omega_n^2\cos(\omega t)$ where $\omega_n = \sqrt{k/m}$, $\zeta = b/(2\sqrt{km})$ and $\delta = F/k$ are the *natural frequency*, *damping ratio* and *static deflection* respectively,

$$x_p(t) = \delta\sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}\cos\left(\omega t + \phi\right) = \delta M\cos\left(\omega t + \phi\right), \quad \tan\phi = \frac{-2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \tag{1}$$

where the square root term and phase angle have values illustrated in Figure 1.

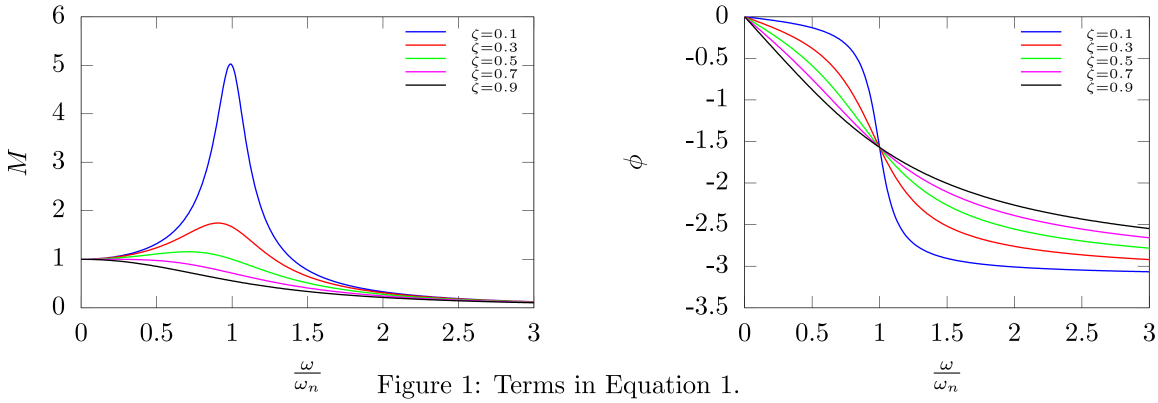


Figure 1: Terms in Equation 1.

For $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \frac{h}{m}\sqrt{(\omega b)^2 + k^2}\cos\left(\omega t + \psi\right)$

$$x_p(t) = h\sqrt{\frac{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}\cos\left(\omega t + \phi + \psi\right) = hM\cos\left(\omega t + \phi + \psi\right), \quad \tan\phi = \frac{-2\zeta\omega\omega_n}{\omega_n^2 - \omega^2} \tag{2}$$

with the terms having the values illustrated in Figure 2.

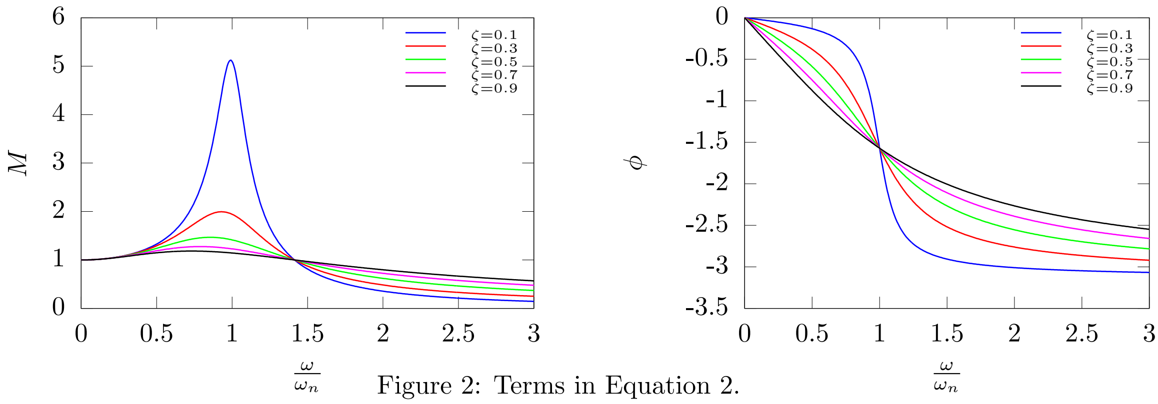


Figure 2: Terms in Equation 2.

1. The way you like to roll is down a sinusoidally bumpy road on your cool unicycle with a suspension, as is illustrated in Figure 3a.

   Unfortunately, the damper (shock absorber) keeps breaking. To diagnose the problem you decide to compute the *damper force* as you are rolling.

   - Determine an expression for the magnitude of the damper force, in terms of k, b, m, λ, v and h. If you choose to do so, you may combine the terms into parameters like ω_n, etc.
   - If you want to decrease the damper force and $\lambda = 100$, $v = 10$, $k = 100$, $m = 100$ and $b = 40$ and can slightly change your speed, should you speed up or slow down? Justify your answer.

2. Consider the usual mass-spring-damper system illustrated in Figure 3b.

   - Determine the total force on the left wall in terms of m, k, b, F and ω.
   - If $m = 2$, $k = 8$, $b = 1/10$ and $\omega = 2.1$:
     - If F is increased slightly, will the force on the left wall increase or decrease?
     - If k is increased slightly, will the force on the left wall due to the damper increase or decrease?

3. Consider an unbalanced motor on top of a mass-spring system (no damping) as illustrated in Figure 3c. As in the homework, the vertical component of the force due to the imbalance is given by $f(t) = m_e \hat{r} \omega^2 \cos \omega t$.

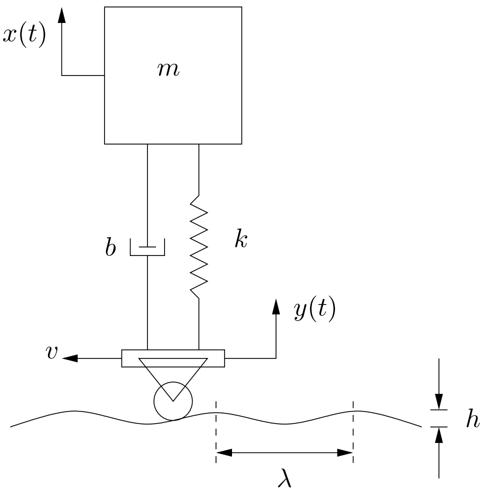
   - Determine an expression for the total force on the mass in steady-state (you may ignore the homogeneous solution) in terms of m, k, m_e, $\hat{r}$ and ω.
   - Manipulate this expression for the total force to be of the form

   $$f_{tot} = -\frac{m_e \hat{r}}{k} M \cos \omega t$$

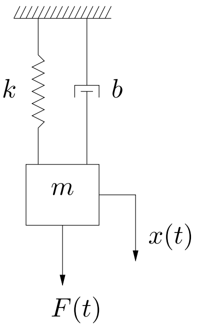
     and plot M versus $\frac{\omega}{\omega_n}$.

4. Consider again the mass-spring-damper system in Figure 3b with $m = 10$, $b = 1/5$, $k = 10$, $F = 10$ and $\omega = 2$.

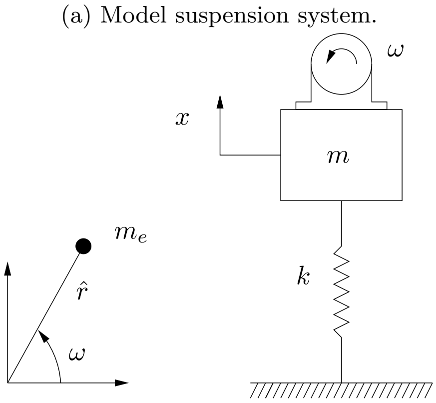
   - What is the magnitude of the steady-state motion?
   - What is the magnitude of the total force on the mass?
   - What is the magnitude of the spring force?
   - What is the magnitude of the damper force?



(a) Model suspension system.



(b) Mass–spring–damper system.



(c) Rotating system with eccentricity for Exercise 3.

Figure 3: Figures for problems.