University of Notre Dame
Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I
Third Exam

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- Do not start or turn the page until instructed to do so. You have 75 minutes to complete this exam.
- This is an open book exam. You may consult the course text and your own course notes, but nothing else.
- There are five problems, each worth 20 points.
- You may **not** use a calculator or other computational aid.
- Your grade on this exam will constitute 15% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in a Blue Book.

Not ignorance, but ignorance of ignorance, is the death of knowledge.

—Alfred North Whitehead

1. Determine the solution to
$$\ddot{x} + 5t^3 x = 0$$
where $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$.

2. Determine a series solution for $(t + 10)\,\ddot{x} + (t + 5)\,x = 0$, $x(0) = 1$ and $\dot{x}(0) = 2$. For what values of t will your series converge to the solution?

3. Consider $\dot{x} + 3x = 2$, $x(0) = x_0$.

   (a) Even though it is not the best way to do it, assume a series solution $\sum_{n=0}^{\infty} a_n t^n$, and find the coefficients.

   (b) For what values of t will this series solution converge to the actual solution?

4. On the next pages are template programs with missing lines. Fill in the blanks with the correct expressions to implement the method indicated. You will receive full credit for filling in only the blank lines. If you need more lines you may do so, but at a somewhat reduced credit. Each program solves the same equation: $\ddot{x} + \frac{1}{t^2+1}x = \cos t$, $x(0) = 1$, $\dot{x}(0) = 2$.

   In the Blue Book, write
   "4(a) 1: *missing stuff on line 1*"
   "4(a) 2: *missing stuff on line 2*" etc.

5. Consider $\dot{x} = t$. What numerical method would be the best to use? Explain your answer including considerations of accuracy as well as the number of computations needed.

Euler's Method

```
      double precision x(2),t,dt

      open(unit=13,file="systemfortran.d")

      dt = 0.001
      x(1) = 0.02
      x(2) = 0.0

      do 10 t = 0, 20, dt
         write(13,*) t,x(1),x(2)
         x(1) = x(1) + _________________________________ (1)
         x(2) = x(2) + _________________________________ (2)
 10   continue
      stop
      end
```

Second-Order Taylor Series Method

```
      double precision x(2),t,dt

      open(unit=13,file="systemfortran.d")

      dt = 0.001
      x(1) = 0.02
      x(2) = 0.0

      do 10 t = 0, 20, dt
         write(13,*) t,x(1),x(2)
         x(1) = x(1) + _________________________________ (3)
         x(2) = x(2) + _________________________________ (4)
 10   continue
      stop
      end
```

Fourth-Order Runge-Kutta Method

```
      double precision x,y,t,dt
      double precision v1,v2,v3,v4,w1,w2,w3,w4
      open(unit=13,file="systemrk4f.d")
      dt = 0.001
      x = 0.02
      y = 0.0

      do 10 t = 0, 20, dt
         write(13,*) t,x,y

         v1 = f(x, y, t)*dt
         w1 = g(x, y, t)*dt

         v2 = _________________________________ (5)
         w2 = _________________________________ (6)

         v3 = _________________________________ (7)
         w3 = _________________________________ (8)

         v4 = _________________________________ (9)
         w4 = _________________________________ (10)

         x = x + _________________________________ (11)
         y = y + _________________________________ (12)
 10   continue
      stop
      end

      double precision function f(x,y,t)
      double precision x,y,t

      f = _________________________________ (13)

      return
      end

      double precision function g(x,y,t)
      double precision x,y,t

      g = _________________________________ (14)

      return
      end
```