University of Notre Dame
Aerospace and Mechanical Engineering

AME 302: Differential Equations, Vibrations and Controls II
Exam 2

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NAME: ______________________________

- You have 50 minutes to complete this exam.
- This is an open book exam. You may consult the course text, your class notes, your own homework sets and any documents provided on the course homepage such as homework solutions, *etc.* You may **not** use a calculator.
- There are three questions. Problems 1 and 2 are worth 30 points and problem 3 is worth 40 points.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page or on the blank pages. If you need more space, use the back of the pages or use additional sheets of paper as necessary.
- Do not start or turn the page until instructed to do so.

"Harry, don't talk like that. As long as I live, the personality of Dorian Gray will dominate me. You can't feel what I feel. You change too often."
"Ah, my dear Basil, that is exactly why I can feel it. Those who are faithful know only the trivial side of love: it is the faithless who know love's tragedies." And Lord Henry struck a light on a dainty silver case, and began to smoke a cigarette with a self-conscious and satisfied air, as if he had summed up the world in a phrase.

Oscar Wilde, "The Picture of Dorian Gray."

1. Solve

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & -\frac{5}{2} \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e^{-4t} \\ \cos 5t \end{bmatrix},$$

where

$$x_1(0) = 1 \quad \text{and} \quad x_2(0) = -1$$

by diagonalization. (30 points)

Hint: if

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \text{then} \qquad A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

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2. Solve

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \sin t \\ 0 \end{bmatrix},$$

where

$$x_1(0) = 0 \quad \text{and} \quad x_2(0) = 0$$

using the method of undetermined coefficients. (30 points)

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3. Consider the system illustrated in Figure 1. The system is comprised of a disk of radius r with mass moment of inertia I. The coordinate x measures the displacement of the disk to the left relative to the unstretched position of the spring with spring constant k that is attached to the disk. Assume that the disk *rolls without slipping.*

   Attached to the center of the disk with a frictionless pin is a massless link of length l. At the end of the link is a mass of mass m. The coordinate θ measures the angular displacement of the link relative to the vertical position. The system acts under the influence of gravity.

   Using x and θ as coordinates for the system, use Lagrange's equations to determine the equations of motion for the system.

(40 points)

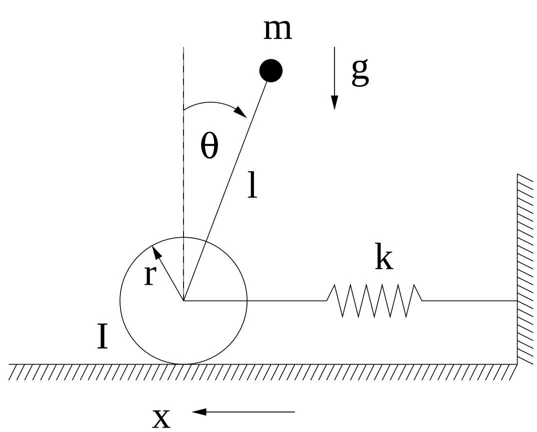

Figure 1. System for Problem 3.

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