University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
Exam 1

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February 21, 2007

NAME: ______________________________

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is a closed book exam. You may only consult one page (two sides) of notes that you have prepared.
- You may **not** use a calculator.
- There are three problems. Problems 1 and 3 are worth 30 points each and problem 2 is worth 40 points. *Show your work* if you want to receive partial credit for any problem.
- Your grade on this exam will constitute 20% of your total grade for the course.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

Some of the world's greatest feats were accomplished by people not smart enough to know they were impossible.

–Doug Larsen

1. (30 points) Consider

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & -1 \\ 0 & 1 & -2 \end{bmatrix}.$$

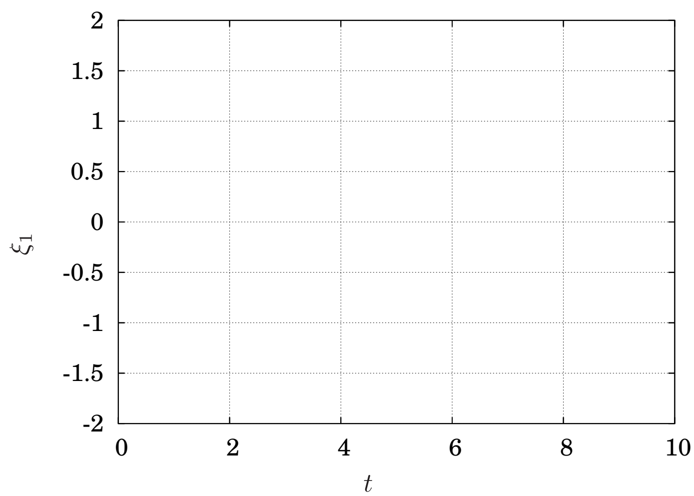
(a) (20 points) Determine the general solution to $\dot{\xi} = A\xi$.

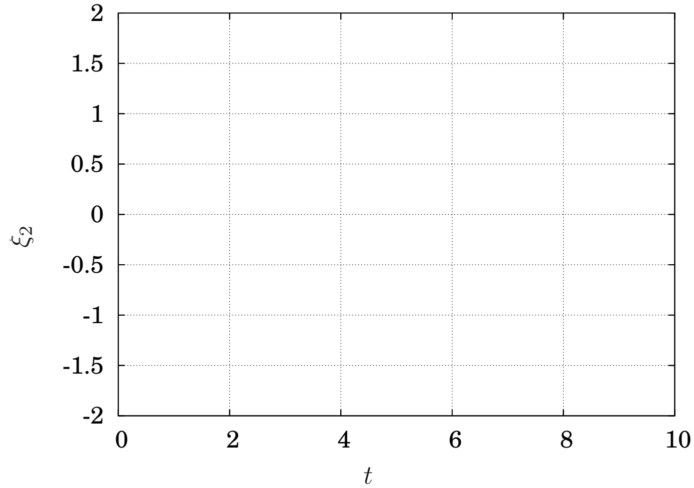
Intentionally left blank.

(b) (10 points) *Sketch* the three components of the solution to problem 1 *versus* time on the following graphs if

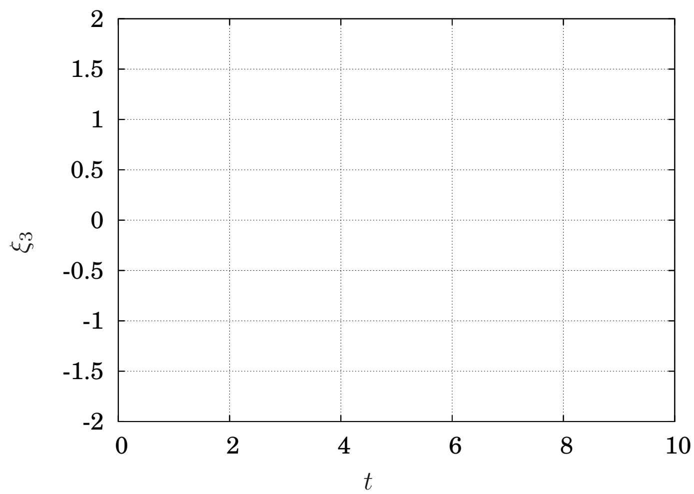
$$\xi(0) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

A qualitatively correct plot will have the correct value at $t = 0$ and the proper long-term behavior. Whether or not, for example, the solution has decayed to $\frac{1}{2}$ of its initial value at $t = 10$ or $t = 20$ does not matter.





2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 2 4 6 8 10
t
ξ_2



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 2 4 6 8 10
t
ξ_3

2. (40 points) Determine the general solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix}.$$

Intentionally left blank.

3. (30 points) Consider $\dot{\xi} = A\xi$ where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

(a) Compute the eigenvalues and eigenvectors.

(b) Write the general solution.