University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
Exam 2

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March 28, 2007

NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 50 minutes to complete this exam.
- This is a closed book exam. You may only consult one page (two sides) of notes that you have prepared, the page of notes you prepared for the first exam and the course handout that had two Laplace transform tables and a table of the properties of Laplace transforms.
- You may **not** use a calculator.
- There are three problems. Problems 1 and 2 are worth 30 points each and problem 3 is worth 40 points. *Show your work* if you want to receive partial credit for any problem.
- Your grade on this exam will constitute 20% of your total grade for the course.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

...consider them both, the sea and the land; and do you not find a strange analogy to something in yourself? For as this appalling ocean surrounds the verdant land, so in the soul of man there lies one insular Tahiti, full of peace and joy, but encompassed by all the horrors of the half known life. God keep thee! Push not off from that isle, thou canst never return!

Herman Melville, *Moby Dick*

1. Consider

$$\ddot{x} + x - x^3 = 0.$$

(a) Write this as two first order ordinary differential equations. (5 points)

(b) Determine all the equilibrium points. (5 points)

(c) Using the Jacobian, determine the differential equation that is the best linear approximation about the equilibrium that is farthest to the right, *i.e.,* about the equilibrium point that has the largest value. (10 points)

(d) Determine the general solution to the linear approximation. (5 points)

(e) Sketch the phase portrait near the equilibrium point. Include the eigenvectors of the Jacobain matrix evaluated at the equilibrium point in the sketch. (5 points)

2. Solve

$$\begin{aligned} \dot{x} - 5x &= \begin{cases} 0 & t < 3 \\ t & 3 \le t < 4 \\ 0 & 4 \le t \end{cases} \\ x(0) &= 0. \end{aligned}$$

(30 points)

3. This problem is going to find the transfer function for a loudspeaker.

   From physics, if a wire of length l carries a current of i amperes and is arranged at a right angle to a magnetic field of strength B tesla, then the force (in Newtons) on the wire is at a right angle to the plane of the wire and magnetic field and has a magnitude

$$f = Bli. \tag{1}$$

   In a speaker, the wire is usually coiled to fit a longer length in a small space.

   This is illustrated schematically in Figure 1. A current, i through the coil, c causes a force, f on the mass (which, in this example, is the magnet) in the direction shown with a magnitude given by Equation 1.

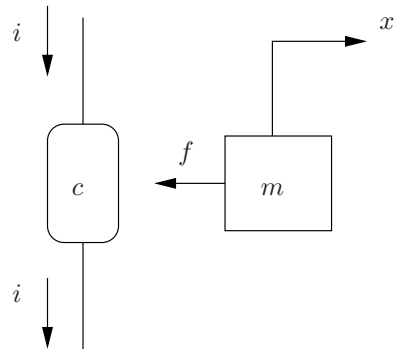

Figure 1. Speaker model for Problem 3.

   Find the transfer function from the current through the speaker coil, i to the location of the mass, x. (20 points)

Now we will attach a high pass filter to the speaker, like the one that is on the tweeters on the cool speakers in my office. This is illustrated in Figure 2.

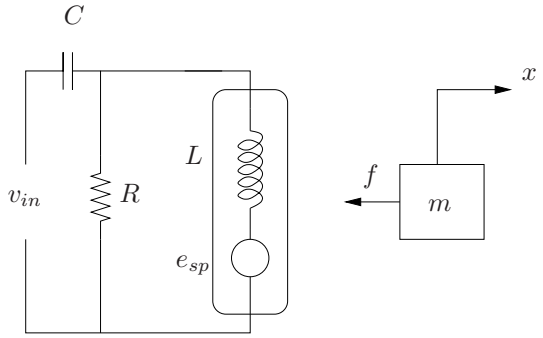

Figure 2. Speaker model for Problem 3.

Everything in the circuit should be obvious except the circle labeled e_{sp}. Just like a d.c. motor, there is a voltage drop across the speaker due to the speaker moving. It is given by

$$e_{sp} = Bl\dot{x}.$$

Find the transfer function from v_{in} to the position of the speaker, x. (20 points)

Hint: the algebra gets messy so 15 of the 20 points will be determining the correct equations that lead to the transfer function. Be sure to clearly indicate with a box around it which equations you are going to use to find the transfer function.