University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
First Exam

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NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 120 minutes to complete this exam.
- You may consult the course text, your class notes, homeworks and homework solutions.
- You may **not** use a calculator or other electronic device.
- There are five problems. Problems 1 and 2 are worth 25 points each. Problem 3 is worth 40 points, Problem 4 is worth nine points and Problem 5 is worth one point.
- If any of your answers involve complex numbers, you must convert the answer to a form that is purely real.
- Your grade on this exam will constitute 20% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

We are going to relentlessly chase perfection, knowing full well we will not catch it, because nothing is perfect. But we are going to relentlessly chase it, because in the process we will catch excellence.

—Vince Lombardi (attributed by Bart Starr)

1. Determine the general solution to

$$\dot{\xi} = A\xi$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 4 & -6 \end{bmatrix}.$$

2. Determine the general solution to

$$\dot{\xi} = A\xi$$

where

$$A = \begin{bmatrix} -5 & 3 \\ -6 & 1 \end{bmatrix}.$$

3. Determine the solution to

$$\dot{\xi} = A\xi$$

where

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -9 & 1 \\ 0 & -1 & -7 \end{bmatrix}$$

if

$$\xi(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

4. In this course we use the null space of matrices all the time. In fact, an eigenvector of a matrix A is in the null space of the matrix $(A - \lambda I)$ if λ is the associated eigenvalue. Also, if λ is an eigenvalue with algebraic multiplicity of m, the generalized eigenvalues of A are in the null space of $(A - \lambda I)^m$.

   Describe in words what a null space is.

5. On a scale from one to 10, state your preference for the second exam in this course. A 10 would indicate that you would strongly prefer to have the second exam in a setting similar to this one: outside of regular class hours and with an extended time limit. A one would indicate that you would strongly prefer the second exam to be in class, 50 minutes and during the regular class hours.