University of Notre Dame
Aerospace and Mechanical Engineering

AME 30315: Differential Equations, Vibrations and Controls II
First Exam

B. Goodwine
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NAME: _____

- Do not start or turn the page until instructed to do so.
- You have 120 minutes to complete this exam.
- You may consult the course text and three pages of written notes.
- You may **not** use a calculator or other electronic device.
- There are four problems, each worth 25 points.
- If any of your answers involve complex numbers, you must convert the answer to a form that is purely real.
- Your grade on this exam will constitute 25% of your total grade for the course. *Show your work* if you want to receive partial credit for any problem.
- Answer each question in the space provided on each page. If you need more space, use the back of the pages or use additional sheets of paper as necessary.

There's nothing noble in being superior to some other man. The true nobility is being superior to your previous self. *–James Raia*

1. Determine the general solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -8 & -6 & 4 \\ -2 & -2 & 0 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}.$$

2. Determine the general solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}.$$

Verify your answer by substituting it into the differential equation.

3. Determine the solution to

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 2 \\ 0 & -1 & -1 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix},$$

where

$$\xi(0) = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$$

4. Determine the solution to

$$\begin{aligned} \dot{x} + x &= \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ -t, & 2 \le t \end{cases} \\ x(0) &= 0. \end{aligned}$$