

# Bisimulation Theory for Multi-Modal Physical Systems

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# Outline

- 1 The CPS context
- 2 Bisimulation of linear DAE systems
- 3 Bisimulation of hybrid systems
- 4 Compositional analysis
- 5 Switching port-Hamiltonian systems
- 6 The state transfer principle
- 7 Selected references

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# Two key aspects of CPS:

- **Hybrid**: integrate continuous and discrete dynamics
- **Complex**: a compositional theory is needed.

Need a theoretical framework for hybrid systems that is compositional.

Notions of interconnection  $\parallel$  to be used:

Interconnection of **physical** systems: sharing of variables

Interconnection of **discrete** systems: sharing of labels

Both are **bilateral**.

**Question**: How to cope with information flow ?

# Topics addressed in this talk:

- **Bisimulation** as a notion of **equivalence**  $\Sigma_1 \sim \Sigma_2$  of hybrid systems.
- Fundamental requirement for any notion of equivalence: for any  $\Sigma$

$$\Sigma_1 \sim \Sigma_2 \quad \Rightarrow \quad \Sigma_1 \parallel \Sigma \sim \Sigma_2 \parallel \Sigma$$

- To be used for exact **model reduction**.
- Notion of bisimulation is key to a 'calculus' of hybrid systems; different representations of the same system.
- One-sided version of bisimulation equivalence: **abstraction** or **simulation**.
- Basis for **verification** and **synthesis** (see comment later on).

# Available notions of equivalence

- Equivalence notions in control theory: equality of transfer functions, state space equivalence.
- Behavioral equivalence: equality of (external) behavior.
- Equivalence for languages: equality of language.
- Notion of equivalence for labeled transition systems: **bisimulation**. Stronger than language equality in case of **non-deterministic** systems.

# Bisimulation of labeled transition systems (automata)

$$(\mathcal{L}, \mathcal{A}, E \subset \mathcal{L} \times \mathcal{A} \times \mathcal{L})$$

A bisimulation relation between two automata  $(\mathcal{L}_i, \mathcal{A}, E_i)$ ,  $i = 1, 2$ , is a subset

$$\mathcal{R} \subset \mathcal{L}_1 \times \mathcal{L}_2$$

with the following property.

Let  $(l_1^-, l_2^-) \in \mathcal{R}$ . Then for every  $a \in \mathcal{A}$  and  $l_1^+ \in \mathcal{L}_1$  such that

$$(l_1^-, a, l_1^+) \in E_1$$

there should exist  $l_2^+ \in \mathcal{L}_2$  such that

$$(l_2^-, a, l_2^+) \in E_2$$

while  $(l_1^+, l_2^+) \in \mathcal{R}$ , and **conversely**.

# Example: Bisimulation of interconnection of labeled transition systems

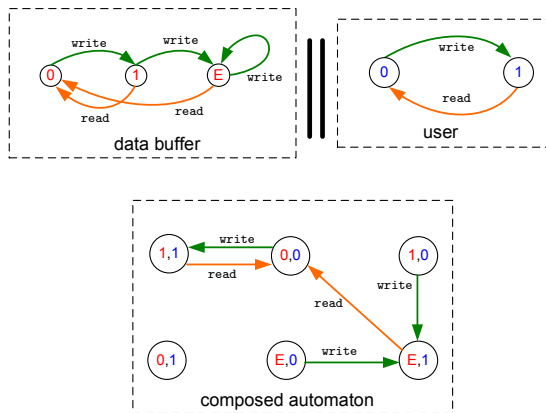
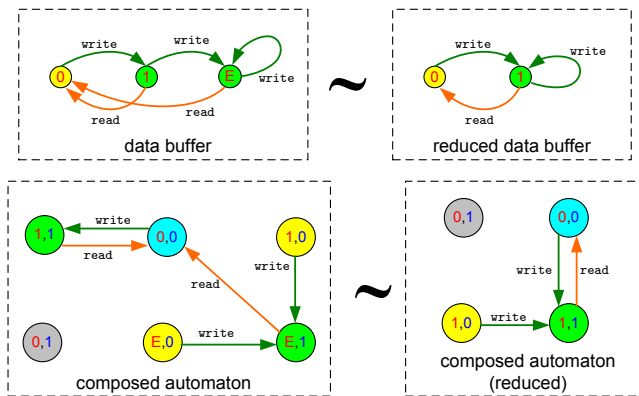


Figure: Interconnection of two labeled transition systems





**Figure:** Bisimulation of buffer and reduced buffer; respectively, of composed system and reduced composed system

Bisimulation of hybrid systems: **extend** the notion of bisimulation for labeled transition systems (automata) to **continuous dynamics**, and then merge.

Different from computer science approaches based on discretization of the continuous dynamics.

Bisimulation theory for continuous dynamics is directly based on the **differential equations** (and not on their solutions) and admits an elegant approach fully using **geometric control theory** and **linear algebra**.

In order to cope with multi-modal physical systems we need a theory of continuous bisimulation for **DAE systems**.

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# Consistent states for linear DAE systems

$$\begin{aligned} E\dot{x} &= Ax, & x &\in \mathcal{X} \\ w &= Hx, & w &= (u, y) \end{aligned}$$

The **consistent subspace**  $\mathcal{V}^*$  for a DAE system  $\Sigma$  is the maximal subspace  $\mathcal{V} \subset \mathcal{X}$  satisfying

$$A\mathcal{V} \subset E\mathcal{V}$$

and equals the set of all initial conditions  $x_0$  for which there exists a continuous solution trajectory of  $\Sigma$  starting from  $x(0) = x_0$ . The space  $\mathcal{V}^*$  is computed as the limit of the sequence

$$\mathcal{V}^0 = \mathcal{X}, \quad \mathcal{V}^j = \{x \in \mathcal{X} \mid Ax = Ev \text{ for some } v \in \mathcal{V}^{j-1}\}, \quad j = 1, 2, \dots$$

# Bisimulation relation

$$\Sigma_i : \begin{aligned} E_i \dot{x}_i &= A_i x_i, & x_i \in \mathcal{X}_i \\ w_i &= H_i x_i \end{aligned}$$

A **bisimulation relation** between  $\Sigma_1$  and  $\Sigma_2$  is a linear subspace

$$\mathcal{R} \subset \mathcal{X}_1 \times \mathcal{X}_2$$

with the following property. Take any pair of consistent states  $(x_{10}, x_{20}) \in \mathcal{R}$ . Then for every trajectory  $x_1, w$  of  $\Sigma_1$  with  $x_1(0) = x_{10}$  there exists a trajectory  $x_2, w$  with  $x_2(0) = x_{20}$  such that  $(x_1(t), x_2(t)) \in \mathcal{R}$  for all  $t \geq 0$ , and **conversely**.

Two systems are **bisimilar**,  $\Sigma_1 \sim \Sigma_2$ , if there exists a bisimulation relation which 'covers all consistent states' of both systems.

## Proposition

Consider two DAE systems  $\Sigma_i$ , with consistent subspaces  $\mathcal{V}_i^*$ ,  $i = 1, 2$ . Denote by  $\pi_i : \mathcal{X}_1 \times \mathcal{X}_2 \rightarrow \mathcal{X}_i$  the canonical projections.

A subspace  $\mathcal{R} \subset \mathcal{X}_1 \times \mathcal{X}_2$  with  $\pi_i(\mathcal{R}) \subset \mathcal{V}_i^*$ ,  $i = 1, 2$ , is a bisimulation relation between  $\Sigma_1$  and  $\Sigma_2$  if and only if for all  $(x_1, x_2) \in \mathcal{R}$  the following properties hold:

- (i) For all  $\dot{x}_1 \in \mathcal{V}_1^*$  such that  $E_1 \dot{x}_1 = A_1 x_1$  there should exist  $\dot{x}_2 \in \mathcal{V}_2^*$  such that  $E_2 \dot{x}_2 = A_2 x_2$  while

$$(\dot{x}_1, \dot{x}_2) \in \mathcal{R},$$

and conversely for every  $\dot{x}_2 \in \mathcal{V}_2^*$  such that  $E_2 \dot{x}_2 = A_2 x_2$  there should exist  $\dot{x}_1 \in \mathcal{V}_1^*$  such that  $E_1 \dot{x}_1 = A_1 x_1$ ,

- (ii)

$$H_1 x_1 = H_2 x_2$$

## Theorem

A subspace  $\mathcal{R} \subset \mathcal{X}_1 \times \mathcal{X}_2$  is a bisimulation relation between  $\Sigma_1$  and  $\Sigma_2$  satisfying  $\pi_i(\mathcal{R}) \subset \mathcal{V}_i^*$ ,  $i = 1, 2$ , if and only if

$$(a) \quad \mathcal{R} + \begin{bmatrix} \ker E_1 \cap \mathcal{V}_1^* \\ 0 \end{bmatrix} = \mathcal{R} + \begin{bmatrix} 0 \\ \ker E_2 \cap \mathcal{V}_2^* \end{bmatrix}$$

$$(b) \quad \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \mathcal{R} \subset \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \mathcal{R}$$

$$(c) \quad \mathcal{R} \subset \ker \begin{bmatrix} H_1 \\ -H_2 \end{bmatrix}$$

The **maximal** bisimulation relation  $\mathcal{R}^* \subset \mathcal{X}_1 \times \mathcal{X}_2$  can be computed by a typical geometric control theory algorithm (similar to the **maximal controlled invariant subspace algorithm**).

$\Sigma_1$  and  $\Sigma_2$  are called **bisimilar** if

$$\pi_i \mathcal{R}^* = \mathcal{V}_i^*, i = 1, 2,$$

where  $\mathcal{V}_i^*$  is the consistent subspace of  $\Sigma_i, i = 1, 2$ .



Generalization of bisimulation theory of **input-state-output** systems  
(Pappas03, vdS04):

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + G_i d_i, & d_i \text{ auxiliary variables for non-determinism} \\ \Sigma_i : & \\ y_i &= C_i x_i, \end{aligned}$$

A bisimulation relation between  $\Sigma_1$  and  $\Sigma_2$  is a subspace

$$\mathcal{R} \subset \mathcal{X}_1 \times \mathcal{X}_2$$

with the following property. Take any  $(x_{10}, x_{20}) \in \mathcal{R}$  and any joint input  $u_1(\cdot) = u_2(\cdot)$ . Then for every  $d_1(\cdot)$  there should exist a  $d_2(\cdot)$  such that the resulting  $x_1(\cdot)$ , with  $x_1(0) = x_{10}$ , and  $x_2(\cdot)$ , with  $x_2(0) = x_{20}$ , satisfy

$$(i) \quad (x_1(t), x_2(t)) \in \mathcal{R}, \quad \text{for all } t \geq 0$$

$$(ii) \quad C_1 x_1(t) = C_2 x_2(t), \quad \text{for all } t \geq 0$$

and conversely.

# Relation with classical equivalence theory of linear systems

## $\Sigma(A, B, C)$ :

- There **exists** a bisimulation relation  $\mathcal{R}$  between  $\Sigma_1(A_1, B_1, C_1)$  and  $\Sigma_2(A_2, B_2, C_2)$  if and only if their **transfer matrices**  $G_i(s) := C_i(Is - A_i)^{-1}B_i, i = 1, 2$ , are the same.
- If  $\Sigma_1$  and  $\Sigma_2$  are **controllable** then  $\Sigma_1 \sim \Sigma_2$  **if and only if** their transfer matrices are the same.
- The bisimulation relation is the graph of an **invertible mapping** if  $\Sigma_1$  and  $\Sigma_2$  are **observable**.
- Factoring by the maximal bisimulation relation between a (non-observable) system **and itself** reduces the system to an equivalent observable system.

# Simulation and abstraction

One-sided version of bisimulation is **simulation**.

## Example

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_1 + x_2$$

$$y = x_1$$

is simulated by ( $\preceq$ )

$$\dot{x} = d, \quad y = x$$

(also called an **abstraction** of the full-order system)

Aggressive technique for model reduction of transition systems; often used for verification purposes.

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## Definition

A hybrid system is described by a six-tuple  $\Sigma^{hyb} := (\mathcal{L}, \mathcal{X}, \mathcal{A}, \mathcal{W}, E, F)$ :

- $\mathcal{L}$  is a set of **discrete states**.
- $\mathcal{X}$  is a finite-dimensional manifold (the continuous state space).
- $\mathcal{A}$  is a set of **discrete communication variables**.
- $\mathcal{W}$  is a finite-dimensional linear space called the space of **continuous communication variables**. Often the vector  $w \in \mathcal{W}$  can be partitioned into an input vector  $u$  and output vector  $y$ .
- $E$  is a subset of  $\mathcal{L} \times \mathcal{X} \times \mathcal{A} \times \mathcal{L} \times \mathcal{X}$ ; a typical element of this set is denoted by  $(l^-, x^-, a, l^+, x^+)$ .  $E$  denotes the **event conditions**.
- $F$  is a subset  $\mathcal{L} \times T\mathcal{X} \times \mathcal{W}$ , where  $T\mathcal{X}$  denotes the tangent bundle of  $\mathcal{X}$ ; a typical element of this set is denoted by  $(l, x, \dot{x}, w)$ .  $F$  denotes the **flow conditions**.

A **hybrid run** of the hybrid system  $\Sigma^{hyb}$  on the time-interval  $[0, T]$  is specified by a five-tuple  $r = (\mathcal{E}, l, x, a, w)$ :

- Discrete set  $\mathcal{E} \subset [0, T]$  denoting the **event times**  $t \in [0, T]$ .
- $l : [0, T] \rightarrow \mathcal{L}$  which is constant on every subinterval between subsequent event times  $t_a, t_b \in \mathcal{E}$ .
- $x : [0, T] \rightarrow \mathcal{X}$ ,  $w : [0, T] \rightarrow \mathcal{W}$ , satisfying for all  $t \notin \mathcal{E}$

$$(l, x(t), \dot{x}(t), w(t)) \in F$$

- A discrete function  $a : \mathcal{E} \rightarrow \mathcal{A}$  such that for all  $t \in \mathcal{E}$

$$(l(t^-), x(t^-), a(t), l(t^+), x(t^+)) \in E$$

## Definition

Consider two hybrid systems  $\Sigma_i^{hyb} = (\mathcal{L}_i, \mathcal{X}_i, \mathcal{A}_i, \mathcal{W}_i, E_i, F_i), i = 1, 2$ .

A **structural hybrid bisimulation relation** between  $\Sigma_1^{hyb}$  and  $\Sigma_2^{hyb}$  is a subset

$$\mathcal{R} \subset (\mathcal{L}_1 \times \mathcal{X}_1) \times (\mathcal{L}_2 \times \mathcal{X}_2)$$

with the following properties:

Take any  $(l_1^-, x_1^-, l_2^-, x_2^-) \in \mathcal{R}$ . Then for every  $l_1^+, x_1^+, a$  for which

$$(l_1^-, x_1^-, a, l_1^+, x_1^+) \in E_1,$$

there should exist  $l_2^+, x_2^+$  such that

$$(l_2^-, x_2^-, a, l_2^+, x_2^+) \in E_2$$

while  $(l_1^+, x_1^+, l_2^+, x_2^+) \in \mathcal{R}$ , and conversely.

## Definition (continued)

Furthermore, take any  $(l_1, x_1, l_2, x_2) \in \mathcal{R}$ . Then for every  $w, \dot{x}_1$  for which

$$(l_1, x_1, \dot{x}_1, w) \in F_1$$

there should exist  $\dot{x}_2$  such that

$$(l_2, x_2, \dot{x}_2, w) \in F_2$$

while  $(\dot{x}_1, \dot{x}_2) \in T_{(x_1, x_2)}\mathcal{R}_{l_1 l_2}$ , and conversely.

(where  $\mathcal{R}_{l_1 l_2} := \{(x_1, x_2) \mid (l_1, x_1, l_2, x_2) \in \mathcal{R}\}$  is assumed to be a manifold).



## Example

If the flow conditions  $F_i$  are described by continuous dynamics

$$\begin{aligned}\dot{x}_i &= A(l_i)x_i + B(l_i)u_i + G(l_i)d_i \\ y_i &= C(l_i)x_i, \quad w_i = (u_i, y_i)\end{aligned}$$

the resets are linear, and the discrete dynamics is independent of the continuous variables, then we can apply the bisimulation theory for continuous systems, and combine this with the standard bisimulation theory for an underlying automaton (Pola, vdS, DiBenedetto, 2006).  
Explicit algorithm for maximal bisimulation relation.

# Critique of bisimulation theory for continuous dynamics

- Bisimulation is a rigid notion, basically combining **state space equivalence** and **reduction to an observable system**.
- **Approximate bisimulation** (Pappas, Girard, Tabuada)
- Theory of bisimulation needs to be extended to systems with **inequality constraints** (cf. Kerber, vdS)

**Note:** Verification for discrete processes seems to be fundamentally different from verification for continuous systems:

In the discrete case both the **system** and the **specifications** can be eventually expressed as an automaton. **Model checking** is then a successful way to do verification for complex systems.

In the continuous case the system and the specifications seldomly can be expressed in the same framework.

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1. Passivity and dissipativity theory
2. Extending ideas from computer science:

Distinguish between two kinds of feedback interconnection:  
**closed** feedback interconnection

$$u_1 = -y_2, y_1 = u_2$$

denoted by  $\parallel_c$ ,  
 and **open** feedback interconnection

$$u_1 = -y_2 + e_1, \quad u_2 = y_1 + e_2$$

with  $e_1, e_2$  external input signals, and denoted by  $\parallel_o$ .

## Proposition

*Simulation  $\preceq$  is compositional with respect to (closed or open) feedback interconnection*

$$\Sigma_{P_1} \preceq \Sigma_{Q_1} \wedge \Sigma_{P_2} \preceq \Sigma_{Q_2} \implies \Sigma_{P_1} \parallel \Sigma_{P_2} \preceq \Sigma_{Q_1} \parallel \Sigma_{Q_2}$$

## Proposition

For *open* feedback interconnection in fact an *equivalence* holds:

$$\begin{aligned} \Sigma_{P_1} \preceq \Sigma_{Q_1} \wedge \Sigma_{P_2} \preceq \Sigma_{Q_2} & \quad (1) \\ \iff \\ \Sigma_{P_1} \parallel_o \Sigma_{P_2} \preceq \Sigma_{Q_1} \parallel_o \Sigma_{Q_2} \end{aligned}$$

This means that for open feedback interconnections, the problem of checking  $\Sigma_{P_1} \parallel_o \Sigma_{P_2} \preceq \Sigma_{Q_1} \parallel_o \Sigma_{Q_2}$  can be reduced to the lower-dimensional problems

$$\begin{aligned} \Sigma_{P_1} & \preceq \Sigma_{Q_1} & (2) \\ \Sigma_{P_2} & \preceq \Sigma_{Q_2} \end{aligned}$$

This is **not** true for closed feedback interconnections.

Which reduction rules **do** hold for **closed** feedback interconnections ?

# Assume Guarantee Reasoning

The **circular** AGR rule is

$$\begin{array}{l}
 (A) \quad \Sigma_{P_1} \parallel_c \Sigma_{Q_2} \preceq \Sigma_{Q_1} \parallel_c \Sigma_{Q_2} \\
 (B) \quad \Sigma_{Q_1} \parallel_c \Sigma_{P_2} \preceq \Sigma_{Q_1} \parallel_c \Sigma_{Q_2} \\
 \hline
 (C) \quad \Sigma_{P_1} \parallel_c \Sigma_{P_2} \preceq \Sigma_{Q_1} \parallel_c \Sigma_{Q_2}
 \end{array}
 \tag{3}$$

## Theorem

*(Kerber, vdS) The circular AGR rule is valid for linear systems.*

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The power variables always appear in conjugated pairs (such as voltages and currents, or generalized forces and velocities), and therefore take their values in dual linear spaces.

### Definition

Let  $\mathcal{F}$  be a linear space with dual space  $\mathcal{E} := \mathcal{F}^*$ , and duality product denoted as  $\langle e | f \rangle = e^T f \in \mathbb{R}$  for  $f \in \mathcal{F}$  and  $e \in \mathcal{E}$ . We call  $\mathcal{F}$  the space of **flow** variables, and  $\mathcal{E} = \mathcal{F}^*$  the space of **effort** variables. A subspace  $D \subset \mathcal{F} \times \mathcal{E}$  is a **Dirac structure** if  $\langle e | f \rangle = 0$  for all  $(f, e) \in D$  and  $\dim D = \dim \mathcal{F}$ .

For the definition of a switching port-Hamiltonian system consider a Dirac structure  $D$  on the space of all flow and effort variables involved:

$$D \subset \mathcal{F}_x \times \mathcal{E}_x \times \mathcal{F}_R \times \mathcal{E}_R \times \mathcal{F}_P \times \mathcal{E}_P \times \mathcal{F}_S \times \mathcal{E}_S$$

Let  $s$  be the number of switches, then every subset  $\pi \subset \{1, 2, \dots, s\}$  defines a **switch configuration**, according to

$$e_S^i = 0, \quad i \in \pi, \quad f_S^j = 0, \quad j \notin \pi$$

We will say that in switch configuration  $\pi$ , for all  $i \in \pi$  the  $i$ -th switch is **closed**, while for  $j \notin \pi$  the  $j$ -th switch is **open**.

For each fixed switch configuration  $\pi$  this leads to the following Dirac structure  $D_\pi$  on the restricted space of flows and efforts

$\mathcal{F}_X \times \mathcal{E}_X \times \mathcal{F}_R \times \mathcal{E}_R \times \mathcal{F}_P \times \mathcal{E}_P$ :

$$\begin{aligned} D_\pi = & \{ (f_X, e_X, f_R, e_R, f_P, e_P) \mid \exists f_S \in \mathcal{F}_S, e_S \in \mathcal{E}_S \\ & \text{such that } e_S^i = 0, i \in \pi, f_S^j = 0, j \notin \pi, \text{ and} \\ & ((f_X, e_X, f_R, e_R, f_P, e_P, f_S, e_S) \in D) \} \end{aligned}$$

Let  $H : \mathcal{X} \rightarrow \mathbb{R}$  denote the total energy at the energy-storage elements with state variables  $x = (x_1, \dots, x_n)$ ; then set

$$\dot{x} = -f_x, \quad e_x = \frac{\partial H}{\partial x}(x)$$

The constitutive relations for the linear resistive elements are given as

$$f_R = -Re_R, \quad R = R^T > 0, \quad (4)$$

implying the power-dissipating property

$$e_R^T f_R = -e_R^T R e_R < 0, \quad \text{for all } e_R \in \mathcal{E}_R, e_R \neq 0$$

The geometric definition of a switching port-Hamiltonian system is given as follows:

### Definition

The switching port-Hamiltonian system is given as

$$\left(-\dot{x}(t), \frac{\partial H}{\partial x}(x(t)), -Re_R(t), e_R(t), f_P(t), e_P(t)\right) \in D_\pi$$

at all time instants  $t$  during which the system is in switch configuration  $\pi$ .

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A particular switch configuration  $\pi$  may entail algebraic constraints on the state variables  $x$ , characterized by the **constraint space**

$$C_\pi := \{e_x \in \mathcal{E}_x \mid \exists f_x, f_R, e_R, f_P, e_P, \text{ such that} \\ (f_x, e_x, f_R, e_R, f_P, e_P) \in D_\pi, f_R = -Re_R\}$$

It follows that

$$\frac{\partial H}{\partial x}(x(t)) \in C_\pi$$

for all time instants  $t$  during which the system is in switch configuration  $\pi$ . Hence if  $C_\pi \neq \mathcal{E}_x$  this imposes algebraic constraints on the state vector  $x(t)$ . Next, we define for each  $\pi$  the **jump space**

$$J_\pi := \{f_x \mid (f_x, 0, 0, 0, 0, 0) \in D_\pi\}$$

## Theorem

$$J_\pi = C_\pi^\perp$$

## Definition (State transfer principle)

Consider the state  $x^-$  of a switching port-Hamiltonian system at a switching time where the switch configuration of the system changes into  $\pi$ . Suppose  $x^-$  is **not** satisfying the algebraic constraints corresponding to  $\pi$ , that is

$$\frac{\partial H}{\partial x}(x^-) \notin C_\pi$$

Then the new state  $x^+$  just after the switching time satisfies

$$x^+ - x^- \in J_\pi, \quad \frac{\partial H}{\partial x}(x^+) \in C_\pi$$

This means that at this switching time an instantaneous jump from  $x^-$  to  $x^+$  with  $x_{\text{transfer}} := x^+ - x^- \in J_\pi$  will take place, in such a manner that  $\frac{\partial H}{\partial x}(x^+) \in C_\pi$ .

Note that the jump space consists of all flow vectors  $f_x$  that may be **added** to the present flow vector corresponding to a certain effort vector at the energy storage and certain flow and effort vectors at the resistive elements and external ports, while remaining in the Dirac structure  $D_\pi$ , **without changing** these other effort and flow vectors. The jump space  $J_\pi$  thus corresponds to a particular subset of conservation laws, and the state transfer principle proclaims that the discontinuous change in the state vector is an impulsive motion satisfying this particular set of conservation laws.

Specialized to switching electrical circuits the state transfer principle is a formalization of the classical **conservation of charge and flux principle**.

### Theorem

*Let  $H$  be a convex function. Then for any  $x^-$  and  $x^+$  satisfying the state transfer principle*

$$H(x^+) \leq H(x^-)$$

*and the resulting system with resets is **passive**.*

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