Control Hierarchies and Tropical Algebras

Jörg Raisch

Fachgebiet Regelungssysteme (Control Systems Group)
Technische Universität Berlin
www.control.tu-berlin.de

&

Systems and Control Theory Group
Max Planck Institute for Dynamics of Complex Technical Systems

Cyber-physical systems are often characterised by large scale, distributed and heterogeneous dynamics. Hierarchical control has been a popular and widely used approach to deal with such systems. It can be interpreted as an attempt to handle complex problems by decomposing them into smaller subproblems and reassembling their solutions into a "functioning" hierarchical structure. It typically involves a number of control layers operating on different time scales that may be clock driven or event driven. Signals on different levels of the control hierarchy may be of different granularity representing phenomena like measurement aggregation when passing from lower to higher level control. In prior work, which is set within Willems' behavioural systems theory (e.g., [7, 8]), we have proposed a formal hierarchical framework that captures the notion of causal signal aggregation (both in time and signal space) for measurements propagating from the plant through the various control layers, and which provides certain consistency conditions that guarantee conflict-free interaction of the different control layers for a given inclusion-type specification [5, 6]. The proposed design procedure is bottom-up, and complexity reduction is achieved by interpreting specifications for lower control levels as abstractions of the plant under low level control. An essential task within the proposed hierarchical control synthesis procedure is then to come up with a suitable choice of specifications for the individual control layers, which, in total, have to be at least as restrictive as the provided overall specification. Because of the dual role of these low-level specifications, this typically involves a non-trivial trade-off. E.g., imposing a less strict specification for a control layer will facilitate the control synthesis task for this layer, but will make the control synthesis task for higher level control more difficult. In this talk, this intuitive trade-off will be formally investigated for a specific scenario: we assume that the top-level controller's sole task is to decide about the timing of certain input events and that all other (discrete and continuous) control inputs are provided by lower control layers. The synthesis of the top-layer controller can then be based on an abstraction of the plant under low-level control in the form of a Timed Event Graph (TEG).

It is a well known fact that the temporal evolution of timed event graphs becomes linear in certain tropical algebras. Tropical algebras, or dioid algebras, are idempotent semirings, and we specifically consider the dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ (e.g., [1]). It can be interpreted as a combination of the well known $(\min,+)$ and $(\max,+)$ algebras; formally, it is defined as a quotient dioid on the set of two-dimensional power series with Boolean coefficients and integer exponents. As there exists a well-developed control theory for systems that are linear in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ (e.g., [4]), synthesis of the top-level controller is straightforward, if an adequate TEG abstraction of the plant under low-level control is available. Uncertainties that are inevitable on an abstraction level can easily be handled by using the dioid of intervals on $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ (e.g. [2]). As dioids are equipped with a natural partial order, there is a corresponding notion of strictness for models in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, and we can use this notion to make the aforementioned trade-off explicit [3].

The contents of this presentation represents joint work with Xavier David-Henriet (TU Berlin), Laurent Hardouin (Université d'Angers), and Thomas Moor (Universität Erlangen-Nürnberg),

References

[1] Baccelli, F., Cohen, G., Olsder, G.J., Quadrat, J.P.: Synchronization and Linearity – An Algebra for Discrete Event Systems. John Wiley and Sons (1992). Web Edition: www-rocq.inria.fr/metalau/cohen/SED/SED1-book.html (2012)

[2] Brunsch, T., Hardouin, L., Maia, C.A., and Raisch, J.: Duality and Interval Analysis over Idempotent Semirings. In: *Linear Algebra and its Applications* 437: 2436-2454 (2012)

[3] David-Henriet, X., Raisch, J., and Hardouin, L.: Consistent Control Hierarchies with Top Layers Represented by Timed Event Graphs. In *Proc. MMAR2012 – 17th Int. Conf. on Methods and Models in Automation and Robotics*, 2012.

[4] Maia, C.A., Hardouin, L., Santos-Mendes, R., Cottenceau, B.: Optimal closed-loop control of timed event graphs in dioids. In: *IEEE Transactions on Automatic Control* 48(12):2284-2287 (2003)

[5] Moor, T., Raisch, J., Davoren, J.: Admissibility Criteria for a Hierarchical Design of Hybrid Control Systems. In *Proc. ADHS03 – IFAC Conference on Analysis and Design of Hybrid Systems*, St. Malo, 2003, pp. 389–394.

[6] Raisch, J., Moor, T.: Hierarchical Hybrid Control of a Multiproduct Batch Plant. In: *Lecture Notes in Control and Information Sciences*, Vol. 322, pp. 199–216. Springer-Verlag, 2005

[7] Willems, J.C.: Models for dynamics. *Dynamics Reported*, 2:172–269, 1989.

[8] Willems, J.C.: Paradigms and puzzles in the theory of dynamic systems. *IEEE Transactions on Automatic Control*, 36:258–294, 1991.