Feedback linearization of the shimmying wheel

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Summary

The classical shimmying wheel is the simplest mechanical model to describe vibration problems in the rolling dynamics of aircraft nose gears, motorcycles, automotive systems and tractor-trailer systems. Such systems can exhibit undesirable unstable rolling motion which can often lead to disastrous results.

The classical shimmy model includes a rigid wheel towed by a horizontal caster attached to a robust cart of constant speed by an elastic king pin. In the presence of dry friction, the single contact point between the wheel and the ground may have a zero velocity, the wheel rolls, and the system is non-holonomic. If the Coulomb friction cannot provide the necessary friction forces for rolling, the wheel slips, and the system is holonomic. There is a non-smooth change between the two kinds of dynamics. Apart of the smooth and symmetric geometrical nonlinearities of the two dynamics, the switches between them provide another strong and important nonlinearity.

The bifurcation analysis of the rolling system proves the existence of a subcritical Hopf bifurcation in the system. Numerical simulation also shows unstable limit cycle for certain parameter values. There exists also an attractor outside the unstable limit cycle which belongs partly to the rolling system, and partly to the sliding one. Energy is introduced into the system via the constant speed of towing, while energy is dissipated at the wheel-ground contact point in case of sliding.

It is difficult to identify the above attractor even with careful numerical simulation. The attractor may look chaotic, and some semi-analytical

investigations support this idea. However, the attractor is quite sensitive for numerical accuracy due to the switches from sliding to rolling. In some respects, it has no sense to improve numerical accuracy further, since the mechanical model by itself is less accurate than its numerical presentation is.

Independently from the exact identification of the strange attractor in the classical shimmy model, it has a practical relevance to control the unstable rolling phenomenon. A simple linear (PD type) controller can stabilize rolling for small deviations only, and fails to do so for large perturbations in the nonlinear system even in the case of pure rolling.

In spite of the complicated algebraic structure of the equations of motion, it is possible to construct a full state feedback linearization control of the rolling system. This controller may serve as a good basis to control shimmy phenomenon via asymmetric braking.

Since the dynamics of slipping is strongly dissipative, the engineering sense expects the feedback linearization control to work perfectly also for the skidding system, where both rolling and slipping occurs. Simulation work shows this is not true when the coefficient of sliding friction is substantionally smaller than the coefficient of static friction.

An alternative approach originated in the feedback linearization is proposed. This alternative control uses the existence of the strange attractor in the skidding system. At least with regard to numerical experimentation, this alternative control strategy seems to be globally stabilizing. The exact mathematical proof of the global stability of the controlled system requires a special construction of Lyapunov function above the phase spaces of the two dynamics. This function has not been constructed so far.

References

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