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Control of Cyberphysical Systems using Passivity and Dissipativity Based Methods

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Abstract

In cyberphysical systems, where compositionality of design is an important requirement, passivity and dissipativity based design methods have shown a lot of promise. Although these concepts are classical, their application to cyberphysical systems poses new and interesting challenges. The aim of this paper is to summarize some of the on-going work in this area by the authors.

I. Introduction

Cyberphysical systems: Cyberphysical systems (CPS) have now emerged as a major research avenue. A precise definition of such systems is difficult to provide both because of the ambitious scope of the applications that are envisaged and because of the fact that the boundaries of this research area are still fluid. Roughly speaking, such systems consist of two components: (i) physical, biological and engineered systems that are usually large scale and complex, and (ii) a cyber core consisting of communication networks and computational availability that monitors, coordinates and controls the physical system. The crucial component is the tight integration between the two components, so that both the analysis and the design of cyberphysical systems has to be done in a joint framework.

Drivers of cyberphysical systems are obvious. As computers become ever-smaller, faster and more efficient, and communication networks become better and ever-cheaper, computing and

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communication capabilities are being embedded in all types of objects and structures in the physical environment. This intimate coupling between the cyber and physical components is being manifested from the nano-world to large-scale wide-area systems of systems; and at multiple time-scales. Research advances in cyber-physical systems promise to transform our world with systems that respond more quickly (e.g., autonomous collision avoidance), are more precise (e.g., robotic surgery and nano-tolerance manufacturing), work in dangerous or inaccessible environments (e.g., autonomous systems for search and rescue, firefighting, and exploration), provide large-scale, distributed coordination (e.g., automated traffic control), are highly efficient (e.g., zero-net energy buildings), augment human capabilities, and enhance societal wellbeing (e.g., assistive technologies and ubiquitous healthcare monitoring and delivery).

However, the tight integration of the physical and the cyber components also renders the development of a systematic analysis and design theory for CPS quite challenging. While various divisions in systems science - control and estimation theory, networking and communication theory, processor and software design - have had tremendous successes over past many decades, these areas have largely developed in isolation from each other. Even the assumptions and models they use may be incompatible. In CPS, a major challenge is to develop a design theory that does not consider the physical and cyber components separately, but as two facets of the same system.

Importance of passivity and dissipativity: A tool that has shown great promise in the design of CPS is that of passivity (and, more generally, dissipativity). Passivity is a classical tool that allows the usage of energy based approaches to general dynamic systems. Originating in circuit analysis, passivity has been applied in multiple domains. The reason that it shows great promise in CPS design is that it allows for compositional design of large-scale systems. Two passive systems in parallel or feedback configuration form a new system that is passive[1]. Since passivity implies other useful properties such as stability, this fact implies that by ensuring that each subsystem is passive, a complex system can be constructed to satisfy certain properties by design and undesirable emergent properties not to arise.

Due to this fact, much attention is now being paid to further develop the theory of passivity and dissipativity, particularly for systems with significant cyber components, both communication and computation. The main aim of this paper is to summarize some of these research directions as

[1]Note that concepts such as Lyapunov stability do not share this property.

pursued by the authors and to present some directions for future work. It should be emphasized that passivity is not the sole answer to all the design challenges inherent in CPS. However, it is an important tool that has shown promise and should be developed to the fullest extent.

Organization of the paper: The paper is organized as follows. We begin with a brief summary of the basic concepts of passivity and dissipativity and their properties in Section II. In Section III we consider the use of passivity in the presence of communication network effects such as delays, quantization, and data loss. We then consider systems with both continuous and discrete dynamics in Section IV. Section V presents some work on the applicability of passivity to some other issues that arise in CPS. Specifically, we consider preservation of passivity levels when the system model is known only approximately and study passivity if the system structure is symmetric. Section VI presents some work on experimental demonstration of passivity based design. Finally, we conclude in Section VII.

II. Passivity and Dissipativity: Basic Concepts

Definitions: Passivity theory has its origins in circuit analysis where systems made up of passive circuit elements were known to be stable and form stable feedback loops [1]. The concept of passivity generalizes the notion of energy by using an 'energy storage function' and an 'energy supply function' that measures the energy input to (and output from) the system. Dissipativity allows the energy supplied to the system to be replaced by a general function of the system input and output [41] (see also the work on QSR dissipativity [17]); thus generalizing the concept of passivity. We begin by defining this more general notion. Consider a nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned} \tag{1}$$

where $x \in X \subset \mathbb{R}^n$ is the state, $u \in U \subset \mathbb{R}^m$ is the control input, and $y \in Y \subset \mathbb{R}^p$ is the output[2]. It is assumed without loss of generality that $f(0,0) = 0$ and $h(0,0) = 0$. Additionally, it is assumed that f is Lipschitz with respect to x. A dissipative system is one that stores and dissipates energy without generating its own energy. The notion of energy supplied to the system

[2]Unless stated otherwise, we consider continuous time systems in this paper. The concepts and results can be developed in analogous fashion for discrete time systems as well.

is captured by an energy supply rate $\omega(\cdot,\cdot)$ where $\omega : U \times Y \to \mathbb{R}$. The energy stored internally can be represented for an individual system through a storage function $V : X \to \mathbb{R}^+$.

Definition 1. *The system (1) is dissipative if there exists a non-negative energy storage function $V(x)$ such that the energy stored in the system is always bounded above by the energy supplied $\omega(u, y)$ to the system over any finite time interval, $T \in [0, \infty)$,*

$$\int_0^T \omega(u, y)dt \geq V(x(T)) - V(x(0)). \tag{2}$$

This form of dissipativity is general, but in practice it may be difficult to identify an appropriate energy supply rate. It may be useful to restrict attention to quadratic energy supply rates.

Definition 2. *A system (1) is QSR dissipative if it is dissipative with respect to the supply rate*

$$\omega(u, y) = y^T Q y + 2y^T S u + u^T R u, \tag{3}$$

where Q, S and R are matrices of appropriate dimensions.

The special case of passivity can now be defined as follows. Passivity is valid for multi-input multi-output (MIMO) systems with the condition that u and y are of dimension m.

Definition 3. *A system (1) is passive if it is dissipative with respect to the following supply rate, for $\delta \geq 0$ and $\epsilon \geq 0$,*

$$\omega(u, y) = u^T y - \delta y^T y - \epsilon u^T u. \tag{4}$$

Additionally, if $\epsilon > 0$ the system is considered to be input strictly passive (ISP) and if $\delta > 0$, it is considered to be output strictly passive (OSP). Finally, if both $\delta > 0$ and $\epsilon > 0$ the system is considered to be very strictly passive (VSP). The nonnegative parameters δ and ϵ are called the passivity levels of the system[3] [18], [47].

Main properties: Passivity and dissipativity offer significant advantages when considering stability and compositionality of dynamical systems.

Theorem 1. *A QSR dissipative system (1) is $\mathcal{L}_2$ stable if $Q < 0$.*

[3]Specifically, $\epsilon > 0$ is the ISP level, $\delta > 0$ is the OSP level and when both $\delta > 0$ and $\epsilon > 0$, (δ, ϵ) are called the VSP levels.

Internal stability can be shown with an appropriate detectability condition, which is a weaker condition than zero-state observability for nonlinear systems [5].

Definition 4. *A system (1) for which* $u(t) = 0\ \forall t$ *is zero-state detectable (ZSD) if* $y(t) = h(x(t), 0) = 0$*,* $\forall t \geq t_0$*, implies* $\lim_{t\to\infty} x(t) = 0$*.*

Theorem 2. *A nonlinear system (1) that is zero-state detectable and QSR dissipative with supply rate (3) is asymptotically stable if* $Q < 0$*.*

For the more general notion of dissipativity (2), asymptotic stability can be shown for systems that are zero-state detectable if $\omega(u, y) < 0$. Passivity provides additional stability and compositionality results beyond those provided by dissipativity theory.

Theorem 3. *A passive system with* $u(t) = 0\ \forall t$ *and a positive definite continuous storage function* $V(x)$ *is Lyapunov stable with Lyapunov function* $V(x)$*.*

The class of passive systems that are output strictly passive are also $\mathcal{L}_2$ stable. Asymptotic stability can be shown for output strictly passive systems that are zero-state detectable.

The use of dissipativity and passivity for compositional analysis of large-scale systems relies on the fact that these properties are preserved in many configurations. Thus, for instance, dissipativity is preserved when two systems are connected in feedback (Fig. 1). Similarly, passivity is preserved when systems are combined in feedback or in parallel (Fig. 2).

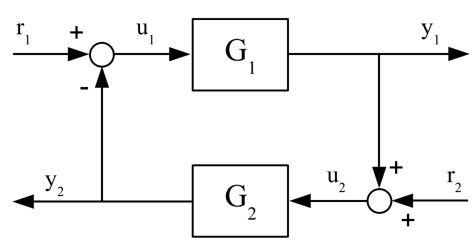

Fig. 1. The general feedback interconnection of two systems.

Combining the stability properties mentioned earlier with these compositional properties yields a powerful method to analyze the stability of a large-scale system. Assuming each component of a system is passive and the components are systematically combined in parallel or feedback,

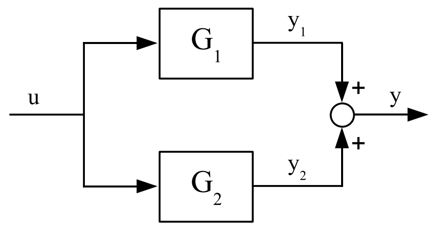

Fig. 2. The parallel interconnection of two systems.

the overall system is automatically passive and stable. Similarly, using dissipativity, results of the following form can be obtained.

Theorem 4. *Consider the feedback interconnection (Fig. 1) of two QSR dissipative systems G_1 and G_2 each with supply rate $\omega_i(u_i, y_i) = y_i^T Q_i y_i + 2y_i^T S_i u_i + u_i^T R_i u_i$, for $i \in [1, 2]$. This interconnection is stable (asymptotically stable) if the following matrix*

$$\hat{Q} = \begin{bmatrix} Q_1 + \alpha R_2 & -S_1 + \alpha S_2^T \\ -S_1^T + \alpha S_2 & R_1 + \alpha Q_2 \end{bmatrix} \tag{5}$$

is positive semidefinite (positive definite) for some $\alpha > 0$.

Passivity Indices: The concept of passivity indices allows the introduction of levels of passivity. This notion is based on earlier work on conic systems [49]. Intuitively, passivity indices provide a measure of the level of passivity in a system such that a negative value indicates a shortage of passivity while a positive value indicates an excess. Two indices, the output feedback passivity (OFP) index ρ and input feedback passivity (IFP) index ν, are needed to fully capture the level of passivity. Under certain assumptions, the class of systems that have passivity indices is equivalent to the class of conic systems [28]. The first index ρ can be defined as follows.

Definition 5. *A system has OFP index ρ if it is dissipative with respect to the supply rate*

$$\omega(u, y) = u^T y - \rho y^T y. \tag{6}$$

A positive or zero value of ρ indicates stability in a system while a negative value indicates instability. Likewise, the IFP index ν can be defined in the following.

Definition 6. *A system has IFP index* ν *if it is dissipative with respect to the supply rate*

$$\omega(u, y) = u^T y - \nu u^T u. \tag{7}$$

The index ν captures the level of the minimum phase property present in a system. A thorough background on passivity indices can be found in [5]. Passivity indices can be defined simultaneously for a given system using a dissipative inequality as follows.

Definition 7. *A system has passivity indices* ρ *and* ν *if it is dissipative with respect to the supply rate*

$$\omega(u, y) = (1 + \rho\nu)u^T y - \rho y^T y - \nu u^T u. \tag{8}$$

As an example of the use of passivity indices in analysis, we state the following result that assesses the stability of the feedback interconnection of two systems.

Theorem 5. *Consider the feedback interconnection (Fig. 1) of two nonlinear systems,* G_1 *and* G_2*. Assume that system* G_1 *has indices* ρ_1 *and* ν_1 *while system* G_2 *has indices* ρ_2 *and* ν_2*. The interconnection is* $\mathcal{L}_2$ *stable if the following matrix is positive definite:*

$$A = \begin{bmatrix} (\rho_1 + \nu_2)I & \frac{1}{2}(\rho_1\nu_1 - \rho_2\nu_2)I \\ \frac{1}{2}(\rho_1\nu_1 - \rho_2\nu_2)I & (\rho_2 + \nu_1)I \end{bmatrix} > 0 \tag{9}$$

This result is valid for MIMO systems assuming that the dimension of the input is the same as the output for both systems in the loop, i.e. from Fig. 1 $u_1, u_2, y_1, y_2 \in \mathbb{R}^m$. The identity matrices in the theorem are also of dimension m. This result can be used to reduce the analysis required to assess stability for the feedback interconnection of two systems. Regardless of the dimension of the system, the dynamics of a system are abstracted to the two parameters of the indices. This is particularly beneficial when the exact dynamics of the systems are unknown, since the passivity indices can be chosen to bound the uncertain behavior. Although presented as an analysis tool, the indices are particularly useful as a control synthesis tool. Given a plant, we can use its passivity indices to obtain bounds on the acceptable indices of a stabilizing controller. A controller can then be designed to satisfy other given objectives while satisfying the design constraints provided by the indices.

Passivity Based Control Design: Passivity has proved useful in control law design [20]. At the most basic level, passivity allows asymptotic stabilization of a plant using *any* negative output feedback. Thus, consider a constant gain output feedback, $u(t) = -Ky(t)$ $(K > 0)$, which yields the passivity inequality $u^T y = -Ky^T y \geq \dot{V}$. With an appropriate detectability assumption, the state of the system will then converge to zero. Notice that this is true even if the system is not fully modeled, as long as the passivity of the dynamics can be guaranteed (for instance, through extensive experiments). Similarly, given a passive plant, any passive controller will stabilize the closed loop system. As an example, PI controllers with positive gains are always passive. This allows flexibility in designing controllers based on criteria other than stability. This is especially useful in switched control when controllers may be chosen for multiple objectives. A supervisory control scheme can monitor the given plant and switch controllers to meet these desired objectives.

Another approach to passivity based design is control around an operating point that is not an equilibrium. While stability is typically studied with respect to the origin, it is often desirable to operate the system around an arbitrary point in space, x^*. The storage function of a passive system can be used to design a control input to shape the energy to produce a minimum at the point x^*. Typically the input is chosen with additional damping so that the state of the system will converge asymptotically to this desired operating point. More detail on energy shaping and other passivity based design methods can be found in [33], [34].

III. Passivity and Dissipativity for Networked Systems

In CPS, an additional complication is the presence of communication networks to transmit data among various components. Communication networks introduce many effects such as delays, data loss, data distortion among others. It is known that such effects can lead to loss of control performance, and even stability. Extending passivity theory for analysis and synthesis, even in the presence of such communication effects, has been a major research direction in recent years [19]. We discuss a few representative works below.

Delays: The most obvious effect introduced by a communication network is to delay any signals transmitted across it. Many traditional results using passivity for analyzing and synthesizing feedback systems are not guaranteed for systems interconnected over a network with delays. The most popular approach for compensating for delay when interconnecting passive systems uses the wave variable transformation [11]. This was first used for telemanipulation

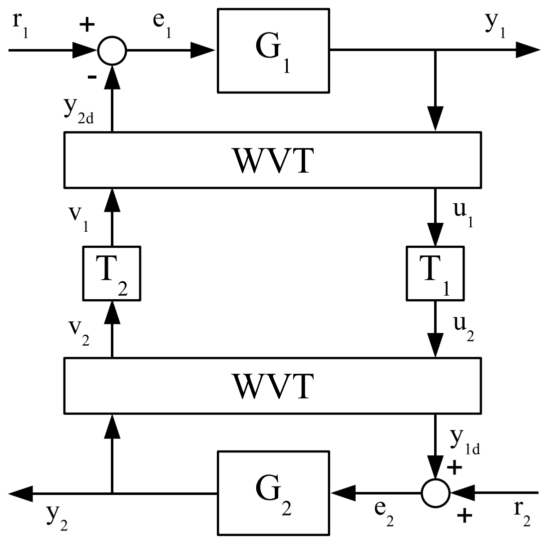

Fig. 3. Use of the wave variable transformation in a network control scheme. The blocks T_1 and T_2 are the time delays in the network. The two blocks labeled WVT are the transformations to wave variables on each side of the network.

systems in [2] and was more recently considered for general networked systems in [21].

Consider the networked control structure given in Fig. 3. System G_1 is the mapping $e_1 \to y_1$ and system G_2 is the mapping $e_2 \to y_2$. Typically one of the two systems is a passive plant and the other is the controller that is designed to be passive as well. In the simplest formulation that we consider, the delays in the network are assumed to be constant although the delays T_1 and T_2 may be different. The received signals are given by

$$u_2(t) = u_1(t - T_1) \tag{10}$$

$$v_1(t) = v_2(t - T_2). \tag{11}$$

The wave variable transformation (WVT) is the input-output coordinate transformation given by

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}, \tag{13}$$

where b is a design parameter. The network can be analyzed in terms of energy storage. The energy going into the network is the square of the two waves u_1 and v_2 while the energy coming out of the network is the square of u_2 and v_1, so that the net energy increase V_N is given by

$$V_N = \frac{1}{2}\int_{t_0}^{t} (u_1^T u_1 + v_2^T v_2 - u_2^T u_2 - v_1^T v_1) d\tau. \tag{14}$$

When the system delays are constant, this expression can be simplified to show that the net energy flowing into the network is positive,

$$V_N = \frac{1}{2}\int_{t-T_1}^{t} u_1^T u_1 d\tau + \frac{1}{2}\int_{t-T_2}^{t} v_2^T v_2 d\tau \geq 0. \tag{15}$$

Even in more general cases, the quantity V_N is always nonnegative so that the network does not generate energy. By the definition of energy stored in the network (14), it can be seen that the energy on the G_1 side of the network bounds the energy on the G_2 side.

$$\frac{1}{2}\int_{t_0}^{T} (u_1^T u_1 - v_1^T v_1) d\tau \geq \frac{1}{2}\int_{t_0}^{T} (v_2^T v_2 - u_2^T u_2) d\tau \tag{16}$$

$$\implies \int_{t_0}^{T} y_1^T y_{2d} d\tau \geq \int_{t_0}^{T} y_2^T y_{1d} d\tau \tag{17}$$

This fact can be used to show stability of the overall system.

Theorem 6. *Consider two nonlinear output strictly passive systems that are interconnected over a delayed network using the wave variable transformation (Fig. 3). If the delays in the network are constant, the interconnected system is $\mathcal{L}_2$ stable.*

Additionally, if the two systems are zero-state detectable, the overall system is asymptotically stable for $r_(t) = r_2(t) = 0$. This approach not only solves the problem of delay for networked passive systems, but can also be used to handle lost data since lost data can be seen as additional energy dissipation. While lost data may hurt performance for passive systems, it does not affect stability.

The presentation so far assumed that the delays are constant. However, typical communication channels have time varying delays. This introduces a problem since energy may be added or removed from time varying delays. One solution to this problem is to modify the wave variable transformation to exactly compensate for energy that is added or removed. This forces the network to be passive and in fact lossless in the case that no data is lost. This assumes that rate of change of the time delays is upper bounded and that the magnitude of the delay is measurable in real time [8]. This approach has also been considered for networking passive switched systems [30].

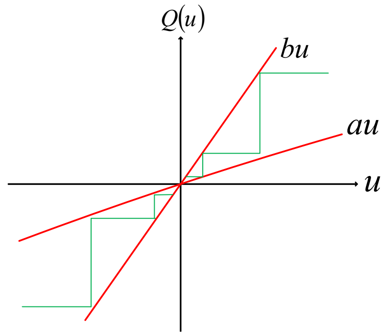

Fig. 4. Sector Bounded Quantizers considered in this paper.

Quantization: Another basic effect introduced by almost every communication channel is the need to quantize any signals that are transmitted across it. Control using quantized feedback has been an important research problem for a long time, see e.g. [6], [10], [12]. However, passivity preservation in the presence of quantization is less well-studied. The work in [51] considered quantizers that are based on the sector bound method (see also [10], [12]) that satisfy

$$au^T u \leq u^T Q(u) \leq bu^T u, \tag{18}$$

where $Q(u)$ is the output of the quantizer with input u and $0 \leq a \leq b < \infty$, see Fig. 4. This kind of quantizers includes several popular quantizers, such as the logarithmic quantizer and the mid-tread quantizer. For these quantizers, results of the following form can be proved.

Theorem 7. *Consider system models as shown in Fig. 5. Assume that system Σ_2 with input u and output y_2 is OSP for $\rho > 0$ and the quantizer Q_i satisfies $a_i u^T u \leq u^T Q_i(u) \leq b_i u^T u$, where $0 \leq a_i \leq b_i < \infty$ and $i = 1, 2$. If a transformation matrix*

$$M \triangleq \begin{bmatrix} m_{11} I_m & m_{12} I_m \\ m_{21} I_m & m_{22} I_m \end{bmatrix}$$

is chosen such that

$$\begin{gathered} m_{21} = 0, \quad m_{11}^2 = 2b_2^2 \\ m_{11} m_{12} = \frac{-b_2^2}{\rho}, \quad m_{12}^2 = \frac{b_2^2 b_1^2}{\rho^2} m_{22}^2, \end{gathered} \tag{19}$$

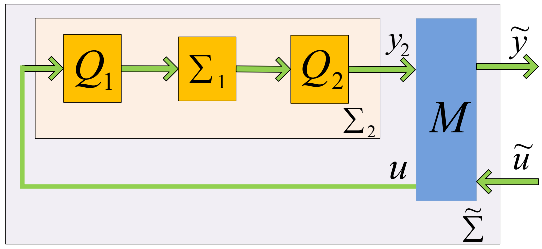

Fig. 5. Input output coordinate transformation used in [51] to preserve OSP level of Σ_1 under quantization.

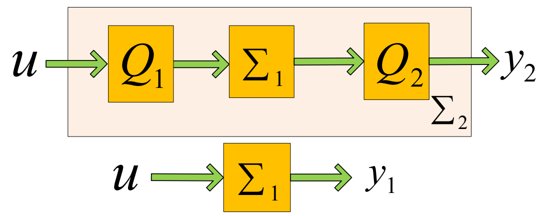

Fig. 6. Quantized-input, Quantized-output system: Σ_1 is $\mathcal{L}_2$ stable and the quantizer Q_i satisfies (18) with $0 \leq a_i \leq b_i < \infty$ for $i = 1, 2$. The control input u and the outputs y_1, y_2 are of the same dimensions.

then the system $\tilde{\Sigma}$ with input $\tilde{u}$ and output $\tilde{y}$ is OSP for $\rho > 0$.

In general, even if system Σ_1 and quantizers Q_i ($i = 1, 2$) are passive, system Σ_2 (the system including input and output quantizers as shown in Fig. 5) may not be passive [48]. However, an appropriate transformation matrix M can be chosen to ensure that the system Σ_2 is passive.

Another direction that was developed in [46] was to investigate whether a quantized system retains any passivity guarantees if the original unquantized system is passive. Results of the following form were shown.

Theorem 8. *Consider system models in Fig. 6, where Σ_2 is quantized version of the system Σ_1. Assume that Σ_1 is $\mathcal{L}_2$-stable, i.e. there exists a $\kappa > 0$ such that $\langle y_1, y_1 \rangle_T \leq \kappa^2 \langle u, u \rangle_T$, $\forall T \geq 0$ and $\forall u$. The quantizer Q_i satisfies $a_i u^T u \leq u^T Q_i(u) \leq b_i u^T u$, where $0 \leq a_i \leq b_i < \infty$. If Σ_1 is ISP for $\nu > 0$, then the following results hold:*

1) *Σ_2 is ISP for $\nu - \kappa(1 + b_1 b_2)$ if $\kappa(1 + b_1 b_2) < \nu$;*
2) *Σ_2 is passive if $\kappa(1 + b_1 b_2) \leq \nu$.*

By setting $a_1 = b_1 = 1$ in the above result, we obtain the case when only the output of Σ_1 is quantized. Likewise, by setting $a_2 = b_2 = 1$, we have the case when only the input is quantized.

Packet losses: Another important effect introduced by communication networks is data loss through causes such as wireless channels, medium access protocols and so on. The data loss can be stochastic or deterministic. Data loss can cause stability and performance loss in control systems. Much work has been done to characterize and mitigate such performance loss.

For passive systems, as discussed above, data loss represents energy dissipation. Hence, while lost data may hurt performance for such systems, it does not impact stability. An interesting formulation was considered by [39] for systems that are not passive, but feedback passive. A feedback passive system requires an external control input to render it passive. Thus, if the control input is transmitted across a network that erases packets, the system may run closed loop at some time steps, but open loop at others. Since the energy dissipation inequality is guaranteed to hold only at the time steps when the control input is received, the overall system may no longer be passive.

The work in [39] considered a discrete time system that evolves as

$$\begin{aligned} x(k+1) &= f\left(x(k), u(k)\right) \\ y(k) &= h\left(x(k), u(k)\right). \end{aligned} \tag{20}$$

The control input $u(k)$ is transmitted across a communication network. At some time steps, the control input is erased, and $u(k) = 0$ is applied. For such systems, passivity was defined as follows.

Definition 8. *A system (20) is passive with initial condition $x(0)$ if there exists a positive definite energy storage function $V(x)$ such that the energy stored in the system is always bounded above by the energy supplied to the system over any finite time interval, $T \in [0, \infty)$,*

$$\sum_{k=0}^{T-1} u^T(k) y(k) \geq V(x(T)) - V(x(0)). \tag{21}$$

Note that this is the discrete version of the original Definition 3. The basic idea of the work is that even if the energy stored in the system is not bounded above by the energy supplied to the system at every time step due to lost packets, the inequality (21) may still hold if the packet loss rate is small enough. This intuition was formalized in the following result. First, we define the concept of feedback passivity.

Definition 9. *A system (20) is feedback passive with initial condition $x(0)$ if there exists a positive definite energy storage function $V(x)$ and a function $\eta(x,u) : X \times U \to X$ that is locally regular such that for any valid control sequence $\{v(k)\}$, the system evolving with the control input $u(k) = \eta(x(k), v(k))$ satisfies the inequality over any finite time interval, $T \in [0, \infty)$,*

$$\sum_{k=0}^{T-1} v^T(k)y(k) \geq V(x(T)) - V(x(0)). \tag{22}$$

Theorem 9. *Consider the system (20). Assume that there exist constants $\xi > 1$ and $0 < \sigma \leq 1$ such that*

$$V(f(x(k),0)) \leq \xi V(x(k))$$
$$V(f(x(k),v(k))) \geq \sigma V(x(k)),$$

where $\{v(k)\}$ is the sequence of inputs that ensures that the output $y(k)$ is identically zero. If for any time T, the ratio $r(T)$ of the time steps at which packets are lost to the time steps at which packets are successfully transmitted is bounded by

$$r(T) \geq \frac{(T-1)\log\xi}{\log\xi - \log\sigma},$$

and the initial conditions $|x(0)| \geq \delta$ and $|v(0)| \geq \delta$ for some $\delta \geq 0$, then the system is locally feedback passive.

This formulation has further been considered for general switched nonlinear systems with both passive and nonpassive modes [40].

IV. Systems with both Continuous and Discrete Representations

Cyberphysical systems have both continuous dynamics due to the physical part and discrete switches due to the cyber component. While a general theory of passivity of such systems is still not available, exciting work is being done in this area.

Sampled data control: The simplest form of systems with both continuous and discrete time dynamics are systems that employ sampled data control. For simplicity, we assume that there is no quantization used. Sampled data control systems arise in CPS both because continuous physical systems are typically controlled by digital devices, as also due to the fact that most communication protocols are disrete time [4], [7]. Consider a continuous-time system Σ_1 with

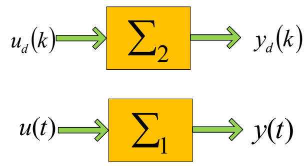

Fig. 7. Sampled-data System with an ideal sampler and a ZOH device, for which $u(t) = u_d(k)$ for $kh \le t < (k+1)h$, $y_d(k) = y(kh)$ for all $k \ge 0$, where h represents the sampling period.

input $u(t)$ and output $y(t)$ and a sampled-data system Σ_2 with input $u_d(k)$ and output $y_d(k)$ as in Fig. 7. A simple discretization method is to use a sampler and a zero-order hold (ZOH) device. For this method, the control inputs for Σ_1 and Σ_2 are related as $u(t) = u_d(k)$ for $kh \le t < (k+1)h$, where h represents the sampling period and the outputs of the two systems are related as $y_d(k) = y(kh)$ for all $k \ge 0$. It is well known that passivity may not be preserved under this discretization scheme (see e.g. [22], [32], [51]). Passivity degradation under standard discretization has been studied in [32] with the following assumption.

Assumption 1. *Suppose for Σ_1, there exists $\alpha > 0$ such that for any $T \ge 0$ and all $u \in \mathbb{R}^m$,*

$$\int_0^T \|\dot{y}(t)\|^2 \mathrm{d}t \le \alpha^2 \int_0^T \|u(t)\|^2 \mathrm{d}t. \tag{23}$$

With this assumption, [32] proved the following result.

Theorem 10. *Consider a continuous-time system Σ_1 and its sampled-data system Σ_2 obtained as described above. Suppose that Assumption 1 is satisfied. The following results hold:*

1) *If Σ_1 is ISP for $\nu > 0$, then its discretization Σ_2 is ISP for $\nu - \alpha h$ for a small enough sampling period h.*
2) *If Σ_1 is VSP for (ρ, ν), then its discretization Σ_2 is VSP for $(\tilde{\rho}, \tilde{\nu})$ for a small enough sampling period h, where*

$$\tilde{\rho} = \rho - h\alpha\rho,$$
$$\tilde{\nu} = \nu - h\alpha - h\alpha\rho - h^2\alpha^2\rho.$$

Similar results were also obtained in [46] for other passivity notions.

Theorem 11. *Consider a continuous-time system Σ_1 and its sampled-data system Σ_2 obtained* *from standard discretization, as shown in Fig. 7. Suppose that Assumption 1 is satisfied.*

1) *If Σ_2 is ISP for $\nu > 0$ and $\alpha h < \nu$, then Σ_1 is ISP for $\nu - \alpha h$;*
2) *If Σ_2 is ISP for $\nu > 0$ and $\alpha h \leq \nu$, then Σ_1 is passive;*
3) *If Σ_2 is VSP for (ρ, ν) and $\alpha^2 h^2 - (\rho - \frac{2}{\rho})\alpha h + \nu^2 - 2 \geq 0$, then Σ_1 is VSP for $(\rho - \alpha h, \nu - \alpha h)$;*
4) *If Σ_2 is VSP for (ρ, ν) and $\rho\nu^2 + \nu - \alpha h \geq 0$, then Σ_1 is passive.*

These results provide bounds on how coarse the discretization interval can be while maintaining passivity. Thus, for instance, if Σ_1 is ISP for $\nu > 0$, from $\alpha h \leq \nu$, we obtain that $\frac{\nu}{\alpha}$ provides an upper bound for the sampling period h for preserving passivity. When α is large (the system may be oscillatory [32]), we need a small sampling period h to ensure passivity.

These results also indicate that input strictly passivity of system Σ_1 may be needed to ensure the passivity of system Σ_2 obtained using a sampler and ZOH. One possible approach to avoid this stricter assumption is to define y_d as

$$y_d(k) = \frac{1}{h} \int_{kh}^{(k+1)h} y(t)\mathrm{d}t,$$

which is called average discretization in [32] (see also [9]). With this signal used as the output of the discretized system, both passivity and passivity levels are preserved after discretization. Another approach used in [22] is to use a passive sampler and hold under which passivity and output strictly passivity can be preserved.

Switched systems: Switched systems are modeled as a family of subsystems with a rule to specify switching among them that determines which subsystem is active at any time. When the rule is not specified, the switching is allowed to be arbitrary. Switched systems represent an important class of hybrid systems where the continuous state must be continuous for all time (see, for example, [23], [24] and the references therein).

There is significant existing work on dissipativity in continuous time [50] and discrete time [25], [40] for switched systems. The concept of passivity indices has also been extended to such systems [29]. Analogous to the use of multiple Lyapunov functions for stability of switched systems, most works consider dissipativity for switched systems by using multiple storage functions. With a different storage function for every mode, the subsystems may be dissipative with respect to a different supply rate. In addition, the notion of cross supply rate is used to capture energy transfer from each active subsystem to each inactive subsystem. A representative result is now provided for dissipativity of switched systems of the form,

$$\begin{aligned} \dot{x} &= f_i(x,u) \\ y &= h_i(x,u), \end{aligned} \tag{24}$$

for $i \in \{1,...,M\}$. For each subsystem i, f_i are assumed to be Lipschitz with respect to x, $f_i(0,0) = 0$, and $h_i(0,0) = 0$. The switching signal $\sigma(t)$ can be used to indicate the active subsystem out of the set $\Sigma = \{1,...,M\}$, i.e. $\sigma : \mathbb{R}^+ \to \Sigma$. Any particular switching instant can be denoted by t_{i_k}, which is the k^{th} time that the i^{th} subsystem becomes active for $i \in \Sigma$. This system becomes inactive at time $t_{(i_k+1)}$ and becomes active again at time $t_{i_{(k+1)}}$. It is assumed that on any finite time interval, t_0 to an arbitrary time T, the system switches a finite number of times K, where K can depend on the value of T. Dissipativity for switched systems is defined in the following [31][4].

Definition 10. *A switched system (24) is QSR dissipative if there exist storage functions $V_i(x)$ bounded by class-K_∞ functions*

$$\underline{\alpha}_i(||x||) \leq V_i(x) \leq \overline{\alpha}_i(||x||), \tag{25}$$

energy supply rates $\omega_i(u,y)$, and cross supply rates $\omega_j^i(u,y,x,t)$ such that the following conditions hold.

1) *Each subsystem i is dissipative with respect to $\omega_i(u,y)$ while active, i.e. for $t_{i_k} \leq t_1 \leq t_2 \leq t_{(i_k+1)}$ and $\forall i,k$,*

$$\int_{t_1}^{t_2} \omega_i(u,y)dt \geq V_i(x(t_2)) - V_i(x(t_1)) \tag{26}$$

where

$$\omega_i(u,y) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q_i & S_i \\ S_i^T & R_i \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}. \tag{27}$$

2) *Each subsystem j is dissipative with respect to $\omega_j^i(u,y,x,t)$ when it is inactive, i.e. for each active subsystem i, $\forall j \neq i$, and for $t_{i_k} \leq t_1 \leq t_2 \leq t_{i_k+1}$,*

$$\int_{t_1}^{t_2} \omega_j^i(u,y,x,t)dt \geq V_j(x(t_2)) - V_j(x(t_1)). \tag{28}$$

[4]Recall that a function $\alpha(x)$ is a class-K_∞ function if $\alpha(0) = 0$, $\alpha(x)$ is non-decreasing for $x > 0$, and $\lim_{x\to+\infty} \alpha(x) \to \infty$.

3) *For all i and j there exist absolutely integrable functions $\phi_j^i(t)$ and inputs $u_i^*(t) = \alpha_i(x(t), t)$ such that $\forall t \geq t_0$ $f_i(0, u_i^*) = 0$, $\omega_i(u_i^*, y) \leq 0$, and*

$$\omega_j^i(u^*, y, x, t) \leq \phi_j^i(t), \forall j \neq i. \tag{29}$$

This notion of dissipativity can be used to show stability of the switched system (24).

Theorem 12. *Consider a QSR dissipative switched system with all subsystems asymptotically zero-state detectable. This system is asymptotically stable if $Q_i < 0$ for all modes i.*

Stability of the feedback interconnection of two dissipative switched systems can also be studied. It is important to note that the feedback interconnection of two switched systems forms a new switched system G. If systems G_1 and G_2 have M_1 and M_2 subsystems, respectively, the new system G may have $M = M_1 \cdot M_2$ subsystems. The set of switching instants of the new system G is the union of the sets of switching instants of the two individual systems. System G_1 is assumed to be dissipative with supply rates $\omega_i^{(1)}$ parametrized by $\{Q_i, S_i, R_i\}$ and G_2 has supply rates $\omega_{\hat{i}}^{(2)}$ with $\{Q_{\hat{i}}, S_{\hat{i}}, R_{\hat{i}}\}$. The following theorem analyzes the active supply rates of the individual systems to establish a dissipative rate of the feedback interconnection.

Theorem 13. *Consider the feedback interconnection of two QSR dissipative switched systems G_1 and G_2. If the supply rates for the two systems satisfy*

$$\hat{Q}_{i\hat{i}} = \begin{bmatrix} Q_i + R_{\hat{i}} & S_{\hat{i}}^T - S_i \\ S_{\hat{i}} - S_i^T & Q_{\hat{i}} + R_i \end{bmatrix} \leq 0, \tag{30}$$

$$\forall i \in \{1, 2, ..., M_1\}, \forall \hat{i} = \{1, 2, ..., M_2\}, \tag{31}$$

the unforced ($r(t) = 0$) feedback interconnection G is stable.

The special case of passivity can also be considered.

Definition 11. *A switched system (24) is passive if it is dissipative with respect to the energy supply rates $\omega_i(u, y) = u^T y - \epsilon_i y^T y$ where $\epsilon_i \geq 0, \forall i$.*

A switched system is considered output strictly passive (OSP) if it is passive with $\epsilon_i > 0$ for all i. Passive switched systems are Lyapunov stable. Asymptotic stability can be shown when negative output feedback is applied or when the system is an output strictly passive switched system.

Theorem 14. *Consider a switched system that is output strictly passive. If all of the subsystems are asymptotically zero-state detectable, then the switched system is asymptotically stable.*

By itself, this result is an indirect method of showing asymptotic stability. There are more direct methods of showing asymptotic stability in the literature. However, using Theorem 14 with Theorem 15 provides open-loop conditions for asymptotic stability of the feedback interconnection of two switched systems.

Theorem 15. *The negative feedback interconnection of two output strictly passive switched systems is again an output strictly passive switched system.*

These two results can be applied together to verify stability of the feedback interconnection of two switched systems. They can also be used to verify stability of large-scale interconnections of passive switched systems. Assuming each component is OSP and the components are sequentially connected in feedback, the large-scale system is passive and stable.

Hybrid systems: The approach of defining dissipativity for switched systems using multiple storage functions can also be applied to hybrid automata. One main difference between switched systems and hybrid systems is that hybrid systems do not necessarily have the same continuous state space in each discrete mode. The state vectors in each mode may be of different dimensions, they may have some states in common, or be entirely different. One class of systems that have been studied in the context of passivity is based on a model of hybrid input-output automata inspired by [26]. The class of systems captured by this model is strictly larger than the class of switched systems considered so far in the paper. It is also more general than classical hybrid automata since it contains both a system input and output. In this class, the continuous dynamics in each discrete mode of the system may be nonlinear but are time invariant. This section represents a summary of this approach, but more detail can be found in [27].

Dissipativity may be defined for this class of hybrid automata using multiple storage functions and multiple supply rates along the lines of the arguments discussed above for switched systems. The main difference is that the supply rate now has two parts: (i) a continuous energy supply rate that is valid when the state trajectory evolves according to continuous dynamics, and (ii) a discrete energy supply rate that is valid at discrete mode changes. The continuous dissipative rate may be different for each mode of the automata.

Since the states in each discrete mode may be different, it is not always clear what a notion of stability such as Lyapunov stability means for such hybrid automata. When we can assume that the state space is the same for every discrete mode and there exists an equilibrium that is valid for all modes, it is possible to show Lyapunov stability for a class of dissipative supply rates. For other hybrid automata, it is possible to show an ultimate boundedness result. Roughly speaking, such a result implies that for some radius B around the origin, the continuous state of the system will be strictly less than B for all hybrid trajectories of the system. This can be shown when the energy supply rates are decreasing when the state is larger than B. The property of dissipativity for hybrid automata is also preserved in feedback. This result along with the stability results, provides simplified conditions for verifying stability of feedback interconnections.

The notion of passivity can be defined as a special case of dissipativity. Under certain conditions on the state space and the equilibria, this notion of passivity implies Lyapunov stability. The property is also preserved when automata are combined in feedback or in parallel. This provides the expected results for the analysis of large-scale systems using passivity theory.

Discrete event systems: As a special case of hybrid systems, it is possible to define dissipativity for discrete event systems (DES). We now mention two directions in this area.

One direction defines dissipativity for DES with respect to the state transition behavior [27]. Since the discrete mode captures the current state of the system, the internally stored energy can be defined with respect to the mode. Transitions are caused by the occurrence of an event which changes the energy level of the system. The energy supply rate can be a function of the current mode and the event that occurs. A definition of this form has a strong connection to an existing notion of Lyapunov stability for DES [35].

An alternative approach studies dissipativity for DES that contain inputs and outputs taking value from a set of symbols. An approach like this is important since the input or output of such a DES can be connected with another DES. This allows for feedback interconnections of DES to be studied using dissipativity. A given DES can even be connected to a dynamical system with an appropriate interface between the two such as a sample and hold with quantization.

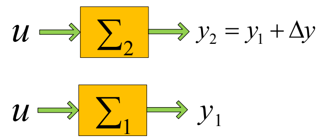

Fig. 8. Illustration of two systems: system Σ_1 with input u and output y_1, system Σ_2 with input u and output $y_2 = y_1 + \Delta y$, where u, y_1, y_2 and Δy are of the same dimensions.

V. Other Challenges in CPS

Interesting work is being carried out to meet many additional challenges posed by CPS, some of which is now summarized.

Approximations: One complication in CPS is that the dynamics of the system may be unknown or may even change with time. To be practical, a design that assumes certain passivity properties for a system block requires some robustness in these properties as the dynamics of the block change. Rigorous characterizations of robustness for the passivity property are needed.

Some representative results are now stated. Consider two system models Σ_1 and Σ_2 as shown in Fig. 8.[5] No assumption on the nature of dynamics for either of the system models Σ_i $(i = 1, 2)$ is made. We can view Σ_2 as an approximation of the system Σ_1 that we are interested in. A commonly used measure for judging how well Σ_2 approximates Σ_1 is to compare the outputs for the same excitation function u [3]. We denote the difference in the outputs by Δy. Note that, in general, Δy will vary depending on the function u. This difference can be due to a host of factors such as modeling error, unmodeled disturbances, or an analysis technique such as linearization, model reduction, and so on.

A reasonable requirement is that the worst case Δy over all control inputs u be small in terms of a suitably defined norm. More formally, we say that Σ_2 is a good approximation of Σ_1 if

[5]In this paper, we focus on system models with 'additive uncertainty', where $y_2 = y_1 + \Delta y$. Similar arguments can be developed for system models with 'multiplicative uncertainty', where $y_2 = y_1(1 + \Delta y)$, see e.g. [5].

there exists a nonnegative constant $\gamma > 0$ such that[6]

$$\langle \Delta y, \Delta y \rangle_T \leq \gamma^2 \langle u, u \rangle_T, \quad \forall u \text{ and } \forall T \geq 0. \tag{32}$$

The value of γ reflects how good the approximation is.

Given this framework, conditions under which the passivity properties of a system Σ_1 can be obtained by analyzing its approximation Σ_2 can now be established [46], [47]. The general result states that if the approximate model Σ_2 can be shown to be input/output/very strictly passive and if the error between the system and its approximation is small, then the original system Σ_1 can be guaranteed to have certain passivity indices. We present the results when the approximate model is input strictly passive and very strictly passive.

Theorem 16. *Consider Σ_1 and Σ_2 in Fig. 8. Suppose that (32) is satisfied for some $\gamma > 0$ and Σ_2 is ISP for $\nu > 0$. Then, the following results hold:*

1) *If $\gamma < \nu$, then Σ_1 is ISP for $\tilde{\nu} = \nu - \gamma$;*
2) *If $\gamma \leq \nu$, then Σ_1 is passive.*

Similarly, suppose that Σ_2 is VSP for (ρ, ν). Then, the following results hold:

1) *If $\gamma < \min\{\rho, \nu\}$ and $\gamma^2 - (\rho - \frac{2}{\rho})\gamma + \nu^2 - 2 \geq 0$, then Σ_1 is VSP for $(\tilde{\rho}, \tilde{\nu})$, where $\tilde{\rho} = \rho - \gamma$ and $\tilde{\nu} = \nu - \gamma$.*
2) *If $\gamma \leq \rho\nu^2 + \nu$, then Σ_1 is passive.*

As can be seen, a better approximation yields a smaller value of γ and means that the passivity properties of a system can be guaranteed even as the dynamics changes, or as a (hopefully simpler) approximation is shown to be passive. In particular, in [46], these methods were applied to approximate models obtained using linearization and model reduction. For instance, for model reduction, a bound on how many modes can be removed while still allowing reasoning about the passivity properties of the original system using the reduced order model was provided.

Symmetric Systems: Symmetry is often exhibited in nature and is related to the concept of a high degree of repetitions or regularities. In [42], agents are categorized into symmetry groups and used to determine stability conditions for large-scale systems. Particularly, stability

[6]Note that (32) implicitly requires the 'error system' with input u and output Δy to be $\mathcal{L}_2$ stable. Particularly, for stable transfer functions G_1 and G_2, if $\|G_1 - G_2\|_{H_\infty} \leq \gamma$, then we can guarantee that (32) is satisfied. In this case, γ is an upper bound on the H_∞ norm of the difference between the transfer functions G_1 and G_2.

for dissipative systems is considered and when subsystems of a symmetric system are dissipative, conditions for stability are derived for the maximum number of subsystems that may be added while preserving stability. These results may be used in the synthesis of large-scale systems with symmetric interconnections.

Dynamics of interconnected nonlinear distributed systems Σ_0, $\Sigma_1, \ldots, \Sigma_m$ are given by

$$\Sigma_i: \quad \dot{x}_i = f_i(x_i) + g_i(x_i)u_i, \quad y_i = h_i(x_i), \quad u_i = u_{ei} - \sum_{j=0}^{m} H_{ij}y_j$$

where $i = 0, \cdots, m$, u_i is the input to subsystem i, y_i is its output, u_{ei} is an external input, and the H_{ij} are constant matrices. If we define $y = [y_1^T, \ldots, y_n^T]^T$, $\widetilde{H} = [H_{ij}]$, and define u, u_e similarly, then the interconnected system can be represented by $u = u_e - \widetilde{H}y$. Using this construction a star-shaped symmetry, for example, can be represented by

$$\widetilde{H} = \begin{bmatrix} H & b & \ldots & b \\ c & h & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \ldots & h \end{bmatrix}$$

where b and c represent the strength of the interconnections between the agents and the following theorem applies to systems with star-shaped symmetries.

Theorem 17. *Consider a $(Q, S, R)-dissipative$ system Σ_0 extended by m star-shaped symmetric $(q, s, r) - dissipative$ subsystems Σ_i. The whole system is asymptotically stable if*

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1}\beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where $\overline{\sigma}$ and $\underline{\sigma}$ are the maximum and minimum singular values of H respectively, $\hat{Q} = -H^T RH + SH + H^T S^T - Q > 0$, $\hat{q} = -h^T rh + sh + h^T s^T - q > 0$ and $\beta = Sb + c^T s^T - H^T Rb - c^T rh$.

The above theorem shows that there exists an upper bound on the number of subsystems that can be added so to preserve stability of dissipative systems and similar results exist for interconnections with cyclic symmetry and chain symmetry. Also extensions on passive systems as a special cases of dissipative systems are studied in [43]. It is important to note that such results are robust under parameter variations. While all subsystems were assumed to be (q, s, r)-dissipative, no restrictions were placed on the actual dynamics.

A further approach is related to Lyapunov stability of symmetric systems. The approach is to define an equivalence class of systems, with each system containing a different number of components and requires us to consider a system with components that define a *group*, G. Interconnections among components may be represented by *generators*, $X \subset G$. For $g \in G$, let $\left\{w_g^{s_1}, w_g^{s_2}, \ldots, w_g^{s_{|X|}}\right\}$ be of outputs, and let $\left\{v_g^{s_1}, v_g^{s_2}, \ldots, v_g^{s_{|X|}}\right\}$ be the inputs. In this more general setting, the dynamics of a component, $g \in G$ are

$$\begin{aligned}\dot{x}_g(t) &= f_g\left(x_g(t)\right) + \sum_{j=1}^{m_g} g_{g,j}\left(x_g(t)\right) u_{g,j}\left(x_g(t), v_g^{s_1}(t), , \ldots, v_g^{s_{|X|}}(t)\right)\\ w_g^s(t) &= w_g^s\left(x_g(t)\right),\end{aligned}$$

for all $s \in X$. A system has *periodic interconnections* if $v_g^s(t) = w_{s^{-1}g}^s\left(x_{s^{-1}g}(t)\right)$, for all $g \in G$ and $s \in X$. Furthermore, if $f_{g_1}(x) = f_{g_2}(x)$, $g_{g_1,j}(x) = g_{g_2,j}(x)$, $w_{g_1}^s(x) = w_{g_2}^s(x)$ and $m_{g_1} = m_{g_2} = m$ for all $s \in X$, $g_1, g_2 \in G$, $x \in \mathbb{R}^n$ and $j \in \{1, \ldots, m\}$, then G has *symmetric components*. Finally, if the control laws also satisfy

$$u_{g_1,j}\left(x_1, w_{s_1^{-1}g_1}^{s_1}(x_2), \ldots, w_{s_{|X|}^{-1}g_1}^{s_{|X|}}(x_{|X|+1})\right) = u_{g_2,j}\left(x_1, w_{s_1^{-1}g_2}^{s_1}(x_2), \ldots, w_{s_{|X|}^{-1}g_2}^{s_{|X|}}(x_{|X|+1})\right) \quad (33)$$

for all $g_1, g_2 \in G$, $j \in \{1, \ldots, m\}$, $s \in X$ and $(x_1, x_2, \ldots, x_{|X|+1}) \in \mathbb{R}^n \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n$ then the system is a *symmetric system on* G. Finally, if two systems are both symmetric with the same generators, they are *equivalent symmetric systems*, and the following Theorem holds [16].

Theorem 18. *Given a symmetric system on a finite group G with generators X, assume there is a function $V_G : \mathcal{D}_G \to \mathbb{R}$ that is smooth on some open domain $\mathcal{D}_G \subset \mathbb{R}^n \times \cdots \times \mathbb{R}^n$ ($|G|$ times) such that 1) V_G may be expressed as the sum of terms corresponding to each component where $V_g : \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{1+|X| times} \to \mathbb{R}$ with*

$$V_G(x_G) = \sum_{g\in G} V_g\left(x_g, x_{Xg}\right) = \sum_{g\in G} V_g\left(x_g, w_{s_1^{-1}g}^{s_1}(x_{s_1^{-1}g}), \ldots, w_{s_{|X|}^{-1}g}^{s_{|X|}}(x_{s_{|X|}^{-1}g})\right),$$

for all $x \in \mathcal{D}_G$, 2) the individual functions corresponding to each component in G are equal as functions, i.e., $V_{g_1} = V_{g_2} = V$ for all $g_1, g_2 \in G$, and 3) for any one of the $g \in G$, $\frac{\partial V_G}{\partial x_g}(x_G)\left(f_g(x_g) + \sum_{j=1}^m g_{g,j}(x_g) u_{g,j}\left(x_g, x_{Xg}\right)\right) < 0$ for all $x_G \in \mathcal{D}_G$. Then $\dot{V}_G(x) < 0$ for all $x \in \mathcal{D}_G$ and for any equivalent symmetric system on $\hat{G}$, there is a $V_{\hat{G}}$ such that $\dot{V}_{\hat{G}} < 0$ on some open domain, $\mathcal{D}_{\hat{G}}$.

The main utility of this theorem is that stability of an entire *class* of systems may be determined by checking only one member of the equivalance class. Current effors are directed toward extending these results to approximately symmetric systems [13]–[15].

VI. Experimental Determination of Passivity Indices

The definition of passivity covered earlier in this paper relies on the existence of an internal representation and an energy storage function. Passivity can be shown when the storage function is non-negative and the inequality (4) is satisfied. An alternative presentation of the results does not rely on an internal representation of the system, only that an input-output condition is satisfied for all inputs of interest [37], [38]. Since this approach does not require an explicit model of the system, it can be used to verify experimentally that a given system is passive without modeling the dynamics and finding a storage function. The following defines this notion of passivity.

Definition 12. *A system is passive with respect to an input set U if there exists a finite constant β that satisfies the following inequality for all inputs $u \in U$ and $\forall T \geq 0$,*

$$\int_0^T y^T(t)u(t)dt \geq -\beta. \tag{34}$$

The constant β captures initially stored energy and may change with different initial conditions. For this definition, the system must satisfy the inequality for all inputs u as well as all finite times T. One issue is that, for passive and non-passive systems, there always exists a finite β to satisfy the inequality for T up to a given time. However, as T goes to infinity, the bound involving a finite β will not hold for non-passive systems. A more practical approach is to test for passivity under zero initial conditions. In this case, the constant β can be taken to be zero,

$$\int_0^T y^T(t)u(t)dt \geq 0. \tag{35}$$

In this case, if the inner product of u and y is negative for any T, the test fails. For systems that are passive with respect to a particular class of inputs, the inner product will vary with the input but will typically grow without bound. If this pattern holds for a sufficiently long time interval, it can be conjectured that the inequality will hold for any T. The length of time that data should be collected to verify the condition is often based on heuristics for a particular application area. This may be based on the desired level of certainty to make a conclusion but also on the system and the particular input being tested.

A similar approach can be taken to estimate passivity indices for a system. In this case the inequality to be satisfied takes the following form,

$$(1+\rho\nu)\int_0^T y^T(t)u(t)dt \geq \rho\int_0^T y^T(t)y(t)dt + \nu\int_0^T u^T(t)u(t)dt. \tag{36}$$

When testing this inequality, the indices can be estimated from each data set and for each time T. One could set $\nu = 0$ and estimate the value of ρ that satisfies (36) as an equality for the particular set of data. When considering all data, there are many values of the indices. There is typically a range of values that satisfy the inequality so some decision must be made on the final set of indices. The calculated indices correspond to the particular set of inputs used.

This approach has recently been applied to an adaptive cruise control algorithm. The parameters of the throttle control system may be adjusted to provide more desirable passivity indices. More on this can be found in [45].

VII. Conclusions

Cyberphysical systems are growing in importance. Passivity and dissipativity have shown great promise in providing systematic design methods for such systems. This paper summarized some ongoing work by the authors in various directions associated with these concepts. Further research needs to be done to fully develop the promise of these methods.

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